

**ABSTRACT**

Report consists of 58 pages, 1 book, 44 references, 2 appendixes.

INVERSE PROBLEMS, STABILITY, INTEGRAL MANIFOLD, STOCHASTIC DIFFERENTIAL EQUATIONS, RANDOM PERTURBATIONS

The object of the investigation are differential and stochastic differential equations, difference equations. The objective of these investigations is to develop methods for solving the inverse problems of differential systems in the presence of random perturbations and to develop mathematical tools of investigation of such problems. Qualitative methods for investigating differential equations, Lyapunov functions method, stochastic differential and integral calculus are used in the investigation. There were obtained the following results:

The inverse problem of stochastic differential systems was solved by the method of Liouville and of Shulgin additional variables. The inverse problem for stochastic Helmholtz systems was solved.

The inverse problem for Birkhoff stochastic systems was solved. The force field was constructed along the given trajectories in the presence of random perturbations.

The set of comparison’s vector functions of the program motion that are independent on time and dependent on time was constructed in the presence of random perturbations. The general problem of constructing the set of stochastic equations of program motion and the set of comparison was solved.

Sufficient conditions for the stability of nonautonomous systems’ program manifold of direct control and of indirect control with stationary and nonstationary nonlinearities was obtained.

The research on stability of the difference dynamical systems by Poisson and by Lagrange was investigated. The research on the Poincaré map near the homoclinic loop was carried out.

The novelty of the results of investigation is the solution of the problems posed under the additional assumption of the existence of random perturbations. The results of the research are theoretical. These results can be used in construction of mathematical models of dynamics of the real processes taking into account the action of random perturbing forces.

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**INTRODUCTION**

The report on the project "Development of methods for solving inverse problems of stochastic differential systems and their application" (2018-2020) contains research on the further development of methods for solving inverse problems of differential systems. The novelty of the topic of this research lies in the further generalization and development of methods for solving inverse problems of dynamics to the class of stochastic differential equations arising in a number of practical applications.

By inverse problems of differential systems we mean both problems of constructing force fields and problems of determining functionals that are stationary in the process of motion, and of restoring and constructing equations of motion of a mechanical system from the given properties of its motion. These problems continue to attract the attention of mathematicians and mechanics for their wide applied potential and go back to such classical inverse problems for differential systems as Newton's problem, Bertrand's problem, Suslov's problem, Meshchersky's problem and Helmholtz's problem. The solutions of these problems with further generalization of their physical interpretation revealed new positions and phenomena in the natural sciences; some of them turned out to be the initial tasks in the formation and development of modern branches of science to control the movements of material systems.

At the present time, possible formulations of inverse problems of differential systems have been formulated and general methods for solving these problems in the class of ODEs have been developed quite fully. At the same time, in many works on stochastic stability and stochastic control, dynamical systems are considered, described by second-order differential equations of the Ito type. These equations describe numerous models of mechanical systems that are important in application, taking into account the effect of external random forces, for example, the motion of an artificial Earth satellite under the action of gravitational and aerodynamic forces, fluctuation drift of a heavy gyroscope in a gimbal, etc. problems of dynamics for the class of stochastic differential equations.

In the modern theory of inverse problems of differential systems, possible formulations of problems are formulated and general methods for solving these problems in the class of ordinary differential equations are developed [1]. And one of the general methods for solving inverse problems of differential systems in the class of ordinary differential equations (the method of quasi-inversion) was proposed by R.G. Mukharlyamov [2]. It should also be noted that the rapidly developing theory of inverse problems of differential systems is a generalization of methods for solving classical inverse problems of dynamics. The main ideas of this theory were first formulated by N.P. Erugin [3], A.S. Galiullin [4, 5] and were further developed in the works of I.A. Mukhametzyanova, R.G. Mukharlyamov and other authors [6, 7-10]. Moreover, for solving inverse problems of differential systems, the initial problem is the construction of differential equations from given integrals, posed by N.P. Erugin [3]. A new stage in the study of inverse problems of ordinary differential systems is associated with the growing interest in the study of the Helmholtz problem in recent years (see, for example, [11, 12] for a survey of works). The classical Helmholtz problem [13] is the problem of constructing equivalent differential equations in the Lagrange form from given ordinary differential equations of the second order. And the equations for which such a transition is possible are called Helmholtz systems [11-15]. The solution of the Helmholtz problem in a particular class of differential equations allows one to extend to this class of equations well-developed mathematical methods of classical mechanics [11, 16, 17].

One of the main requirements in the theory of inverse problems of differential systems associated with the system's operability and its intransigence to disturbances is the requirement of stability of the given properties of motion [5], therefore, solving the problem of stability of programmed motion [18-24] is essential for the further development of the qualitative theory inverse problems of differential systems and the theory of constructing systems of programmed motion. The foundations and methods for solving inverse problems of differential systems have been developed mainly only for deterministic systems, the equations of which are ordinary differential equations. And in most cases, ordinary differential equations are a fairly effective apparatus for modeling real processes in dynamic systems. But the increased requirements for the accuracy and performance of material systems leads to a situation where many observed phenomena can no longer be explained from the standpoint of deterministic processes. This circumstance requires, in particular, the involvement of probabilistic laws for modeling the behavior of real systems. Therefore, the problem of generalizing the methods for solving inverse problems of differential systems to the class of stochastic differential equations seems to be urgent. In this project it is assumed that the dynamics of the system under study is described by an ordinary differential equation, and fluctuations of its parameters are processes of the "white noise" type, the intensity of which may depend on the state of the system. A random process, caused only by initial deviations and fluctuations of parameters, is, as is known, Markovian [25], and the equation describing its trajectory can be understood as the stochastic differential Itô equation. Stochastic differential equations of the Ito type describe numerous models of mechanical systems that are important in the application, taking into account the effect of external random forces, for example, the motion of an artificial Earth satellite under the action of gravitational forces and aerodynamic forces [26], or fluctuation drift of a heavy gyroscope in a gimbal [27] and a lot others.

In this paper, we study inverse problems of dynamics in a probabilistic formulation, investigate the influence of random perturbing forces on the solvability of inverse problems of dynamics and the stability of the given properties of motion. Within the framework of this report, the results of studies on the construction of a set of stochastic differential equations with a given stable integral manifold are presented. This report reflects research on the topic "Development of methods for solving inverse problems of stochastic differential systems and their application" for 2018-2020. The results obtained by the performers are new and make a significant contribution to the modern qualitative theory of differential equations, as evidenced by the publication of the works of the performers in authoritative mathematical journals and their participation with scientific reports at international conferences.

For 2018, 2019 years of the reporting period 2018-2020, interim reports on research work "Development of methods for solving inverse problems of stochastic differential systems and their application" were prepared, inventory numbers No. 0218РК00090, No. 0219РК00051.

**MAIN PART** **OF THE REPORT ON SCIENTIFIC-RESEARCH WORK**

**1 Inverse problems in the class of stochastic differential equations**

**1.1 The Liouville additional variables method in the stochastic Helmholtz problem**

Using the Liouville method, first-order stochastic ito equations are reduced to stochastic equations of canonical structure.

Let us consider a system of first-order stochastic ito differential equations

 (1.1.1)

It is necessary to bring the system of equations (1.1.1) to equivalent equations of the Hamiltonian structure.

We assume that the functions included in the above equations have the smoothness necessary for further reasoning and satisfy the existence and uniqueness theorem for solving the Cauchy problem in the class of stochastic differential ito equations [28].

Here are systems of random processes with independent increments, which, following [28], can be represented as a sum of processes  Here  is a vector Wiener process. is a Poisson process. is the number of jumps of the process  on the interval [0, t] that fall on the set . is a vector function that maps the space into the space of process values  for any t. And the equivalence of solutions of equations is understood in the sense of equivalence almost certainly [29]: random processes and are equivalent almost certainly or in a vector-wise way, if the  follows almost certainly from for all .

This problem in the absence of random perturbations  is considered in [30].

Here and further, summation is assumed for repeated indices of multipliers. Indexes change from 1 to , and the index changes from 1 to .

We use the Liouville method [30] for solving this problem. Namely, we introduce auxiliary variables , and we define the Hamilton function of the extended system as

 (1.1.2)

Then and the corresponding equations of motion of the extended system are written as

 (1.1.3)

where the equations coincide with the original equations (1.1.1), and the equations are used to determine auxiliary variables .

Theorem 1.1.1 Necessary and sufficient conditions for the representation of the first-order Ito equation (1.1.1) in the form of equations of the canonical structure (1.1.3) are the representation of the Hamilton function in the form (1.1.2) using additional variables  that are determined from the equations (1.1.3a).

**1.2 Shulgin's method of additional variables in the inverse problem of stochastic differential systems**

Using the Shulgin method, second-order stochastic Ito equations are reduced to stochastic equations of the Lagrangian structure.

Let be a system of second-order stochastic Ito equations of the form

 (1.2.1)

It is necessary to reduce the system of equations (1.2.1) to equivalent equations of the Lagrangian structure.

Here is a system of random processes with independent increments, and the equivalence of solutions of equations is understood in the sense of equivalence almost certainly.

To solve this problem, we will use the Shulgin method [31]. Namely, we introduce additional variables and define the Lagrange function of the extended system as

 (1.2.2)

here . Then  and, therefore, equation (1.1) will be equivalent to the equation

  (1.2.3a)

And to determine the auxiliary variables we write the following equation of the Lagrangian structure

  (1.2.3b)

here  are some arbitrary functions that are continuous in their arguments and do not violate the conditions of the existence and uniqueness theorem of the Cauchy problem for the system of equations (1.2.3a) and (1.2.3b) [28], and in particular can be chosen to be identically equal to zero for all  and .

Set of equations (1.2.3a) and (1.2.3b)

 (1.2.3)

forms a system  of Lagrangian structure equations with a generalized Lagrangian .

Theorem 1.2.1 Let the functions   included in equations (1.2.1) have continuous partial derivatives of the second order (including mixed ones) and, in addition, have continuous partial derivatives of the third order 

Then sufficient conditions for representing the system of stochastic equations (1.2.1) in the form of Lagrangian structure equations (1.2.3) are the representation of the Lagrangian in the form of (1.2.2) using additional variables that are determined from equations (1.2.3b).

**1.3 Inverse problem for stochastic Helmholtz systems**

The classical Helmholtz problem [13] is the problem of constructing equivalent Lagrange equations of motion for a mechanical system in the form of Newton using the given equations of motion. The results of further research of the Helmholtz problem can be found in [16, 17, 32], which, along with their own research, mainly in the class of odes (ordinary differential equations) and dstp (partial differential equations), provides a historical overview of the development and generalization of this problem.

Solving the Helmholtz problem [13] in a particular class of differential equations allows us to extend well-developed mathematical methods of classical mechanics to this class of equations.

The second-order stochastic ito equations are used to construct stochastic Lagrangian structure equations that are equivalent in the sense of almost certainly. Conditions for direct and indirect analytical representation of the Lagrangian in the presence of random perturbations are determined. The results are illustrated with examples.

Let a system of stochastic second-order Ito differential equations be given in the form

(1.3.1)

It is necessary to bring the system of equations (1.3.1) to equivalent equations of the Lagrangian structure

 (1.3.2)

We assume that the functions included in the above equations have the smoothness necessary for further reasoning and satisfy the existence and uniqueness theorem for solving the Cauchy problem in the class of stochastic differential Ito equations [28].

Here  are systems of random processes with independent increments, which, following [33], can be represented as a sum of processes:  where , is a vector Wiener process; is a Poisson process; is the number of jumps of the process in the interval [0, t] that fall on the set ;  is a vector function that maps the space  into the space of process values  for any *t*, and the equivalence of solutions of equations is understood in the sense of equivalence almost certainly (n. n.) [29]: random processes  and  are equivalent to n. n. or in a vector-wise way, if from  n. n. follows  for all  almost certainly.

This problem in the absence of random perturbations  is considered in [16].

Here and further, summation is assumed for repeated indices of multipliers. Indexes  change from 1 to , and the index  changes from 1 to .

Fair

Theorem 1.3.1 In order for the equation (1.3.1) to admit an equivalent analytical representation (1.3.2), it is necessary and sufficient to meet the conditions

   (1.3.3)

where 

Remark 1.3.1 If  then the conditions (1.3.3) coincide with the conditions of R.M. Santilli [16].

The above problem can be solved in a slightly different way if we use a method similar in the case of ordinary differential equations to the method described in the book by E. T. Whittaker [30] and going back to the works of N. Ya. Sonin [34] and Zh. Darboux [35].

The follow theorem takes place

Theorem1.3.2 In order for the equation (1.3.1) to admit an equivalent analytical representation (1.3.2) it is necessary and sufficient to meet the following conditions

  (1.3.4)

Remark 1.3.2 If  the conditions (1.3.4) coincide with those of E.T. Whitaker [30].

In particular, when  the conditions (1.3.3) for the transition from (1.3.1) to (1.3.2) take the form (1.3.5)

 (1.3.5)

and the conditions (1.3.4) take the form (1.3.6), respectively

 (1.3.6)

**2 Construction of equations of mechanics for a given integral manifold in the presence of random perturbing forces**

**2.1 Stochastic Helmholtz problem for Birkhoff systems**

According to the given stochastic Langevin-Ito equation in an indirect representation, both the Hamiltonian structure equation and the Birkhofian structure equation are constructed. The method of moment functions is used to determine a functional that takes a stationary value on solutions of a given stochastic Birkhoff equation. The results are illustrated with an example.

According to the equation given in the Langevin-Itô form

 (2.1.1)

it is required to construct an equivalent equation of the Hamiltonian (or Birkhofian) structure.

Here , where, following [22], is a Wiener process, is a Poisson process,  is the number of jumps of the process  in the interval  falling on the set where 

This formulation of the problem in the absence of random perturbations () was considered by R.M. Santilli [16].

To solve the problem, we first introduce a new variable and rewrite the given equation (2.1.1) in the form

 (2.1.2)

And then, using substitutions





we rewrite equation (2) as

 (2.1.3)

Further, the stochastic equation of the Hamiltonian structure

 (2.1.4)

by substitution



and matrices



and also taking into account that



rewrite as

 (2.1.5)

Or, if we introduce the inverse matrix for 



and -dimensional vector



then equation (2.1.5) transforms to the equivalent equation

 (2.1.6)

Construction of the Hamiltonian in the indirect representation. Consider the problem of indirect representation of equation (2.1.3) in the form of an equation of Hamiltonian structure (2.1.6), i.e. using some matrix , consider the relation

 (2.1.7)

or

 (2.1.7 ')

where .

For identity (2.1.7) to hold, the conditions

 (2.1.8)

 (2.1.9)

 (2.1.10)

From (2.1.9) and (2.1.10) follows that the equality

 (2.1.11)

takes place.

Therefore, it is true

Theorem 2.1.1. For an indirect representation of the stochastic equation (2.1.3) in the form of the stochastic Hamilton equation (2.1.6), it is necessary and sufficient to satisfy conditions (2.1.8), (2.1.10), (2.1.11).

Further, in Section 2.1, the following stochastic Helmholtz problem is considered: for a given equation

 (2.1.12)

construct a stochastic equation of the Birkhofian structure of the form

 (2.1.13)

Here  is called the Birkhoff function, and  is called the Birkhoff tensor [39] with components  =   .

The Helmholtz problem in the class of Langevin-Ito stochastic differential equations is divided into two interrelated problems. At the first stage, a stochastic analogue of the Lagrange, Hamilton, or Birkhoff equations is constructed using a given equation. And then, at the second stage, using the constructed, ,  or ,  we need to construct the required functional (action according to Hamilton or according to Birkhoff). Therefore, this section also considers one of the options for constructing a stochastic analogue of the Birkhoff action. And the functional that takes a stationary value on the solutions of equation (2.1.13) is constructed in the form of the averaged action according to Birkhoff in the form 

**2.2 The problem of constructing a force field on given trajectories (independent of velocities) in the presence of random perturbations**

Let the trajectory

Λ: λ (x, y) = 0. (2.2.1)

is given.

It is required to construct a force field in the presence of stochastic disturbing forces, so that the constructed force field has a given trajectory as an integral manifold

(2.2.2)

The projections of the velocity of a material point onto the coordinate axes are determined from the equation

Differentiating the last expression with respect to time, we obtain

We introduce the vector function *A* and the Erugin matrix *B*

Here such that Here

Definition 2.2.1 We say that some function  from the class  if it is continuous in  and Lipschitz in the whole space and satisfies the condition of linear growth in:  with some constant 

Let then

a) if for any then

(2.2.3)

for arbitrary from class *K*;

b) if for any then

(2.2.4)

for arbitrary from class *K*.

Theorem 2.2.1 In order for the set of force fields (2.2.2) has a given trajectory (2.2.1) in the presence of random perturbations from the class of processes with independent increments, it is necessary and sufficient that one of conditions (2.2.3), (2.2 .4) be satisfied.

**2.3 The problem of constructing a force field along given trajectories (depending on velocities) in the presence of random perturbations**

Let us consider the case when λ depends on both generalized coordinates and generalized velocities

(2.3.1)

It is required to construct a force field in the presence of stochastic disturbing forces, so that the constructed force field has a given trajectory as an integral manifold

(2.3.2)

Differentiating (2.3.1) with respect to time, we obtain On the other hand

We introduce the vector function and the Erugin matrix

her such that the relation takes place. Here

Let then

a) if for any then

(2.3.3)

for arbitrary from class

b) if for any x, y, then

(2.3.4)

for arbitrary from class

Theorem 2.3.1 In order for the set of force fields (2.3.2) has a given trajectory (2.3.1) in the presence of random perturbations from the class of processes with independent increments, it is necessary and sufficient that one of the conditions (2.3.3), (2.3.4) be satisfied.

**3 On construction of the comparison function of program motion in probable statement**

One of the stochastic inverse problems of differential systems is considered. It is the problem of constructing both a set of first-order stochastic differential equations, and a set of comparison functions. There is stability in probability of the given program motion with respect to comparison functions.

It is required to construct the corresponding set of equations of motion for the material system

, , (3.1.1)

by the given program of motion

, , , . (3.1.2)

Let the considering equations be the class of equations admitting the existence of a unique up to stochastic equivalence of solution of the equation (3.1.1) with initial condition . We also assume that there is a set of n-dimensional vector functions. is holomorphic vector functions in some -neighborhood  of the integral manifold (3.1.2) for all . There is stability in probability of the program motion (3.1.2) with respect to.

Following [1] the equation of perturbed motion of the material system, for which the given motion (3.1.2) is possible, is represented as

. (3.1.3)

Here is a vector function and is a -dimentional Erugin type matrix such that , .

In the future we need the following definitions:

Definition 3.1.1 [36]. A function  is called the function of Khan class  if it is continuous and strictly increasing and satisfies the condition .

Definition 3.1.2 [25]. The program manifold (3.1.2) of the equation (3.1.1) is called -stable in probability if



**3.1 A set of vector functions that are not externally depend on time**

Theorem 3.1.1 Let there exist a Lyapunov function  on the neighborhood  of the integral manifold satisfying the conditions

, , (3.1.4)

, . (3.1.5)

Then the program motion  of system (3.1.3) is asymptotically - stable in probability with respect to an arbitrary - dimensional vector function , which is continuous on the neighborhood  for .

Proof. By definition of stability [1] we consider the difference . By the condition of the theorem there is a Lyapunov function with properties (3.1.4), (3.1.5). This provides an asymptotically - stability in probability of program motion  [37]. i.e.

. (3.1.6)

And from the continuity of the vector function  and conditions (3.1.6) we have

.

This means that the motion  of the system (3.1.3) is asymptotically stable with respect to the vector function .

**3.2 A set of vector functions  depending on and **

Theorem 3.2.1 Let there exist a Lyapunov function  on the neighborhood  of the integral manifold  with properties (3.1.4), (3.1.5).

Then the program motion  of system (2.3) is asymptotically - stable in probability with respect to an arbitrary - dimensional vector function, which is continuous in and  and satisfying the condition

, . (3.1.7)

Here .

Proof. The existence of a function  with properties (3.1.4), (3.1.5) implies an asymptotical uniform in  -stability in probability of the motion  [37]

.

And we get from (3.1.6) and (3.1.7) that

.

Consequently, there is an asymptotic stability in probability of the motion  of the system (3.1.3) with respect to the vector function .

**3.3 Program motion and a set of comparison vector functions **

Let the program motion be given as

. (3.1.8)

Here , , .

Suppose that it takes place for all,  on the neighborhood 

, . (3.1.9)

The set of equations of the perturbed motion for which the given program (3.1.8) is one of the possible ones, can be represented as

. (3.1.10)

Here is a vector function and is a -dimentional Erugin type matrix such that , .

Consider the continuous -dimensional vector functions  satisfying the condition

, . (3.1.11)

Here , .

Theorem 3.3.1 Let there exist a Lyapunov function  on the neighborhood (3.1.9) of the integral manifold (3.1.8) satisfying the condition (3.1.4) and

, . (3.1.12)

Then the integral manifold (3.1.8) is asymptotically stable in probability with respect to an arbitrary - dimensional vector function , which is continuous in  and  and satisfying condition (3.1.11) for .

Let us consider a set of - dimensional vector functions of the form . Let the equation of perturbed motion (3.1.3) in the first approximation has the form

. (2.13)

Let us consider the Lyapunov function  and - dimensional vector function. Here . In this particular case  has the form

. (3.1.14)

Also suppose that

1) the matrix is definitely negative and the vector function satisfies the condition ;

2) the matrix is continuous and limited for all .

Then from properties 1), 2) and Theorem 3.2.1 the following theorem holds.

Theorem 3.4.1 Let and are continuous matrices such that conditions 1) and 2) hold. Then the motion of the system (3.1.3) is stable in probability with respect to an arbitrary vector functions .

**4 Stability of the program manifold of automatic control systems with variable coefficients**

**4.1 Stability of the program manifold of non-autonomous direct control systems with stationary nonlinearities**

The absolute stability of the program manifold of non-autonomous basic control systems with stationary nonlinearities is investigated.

The stability conditions for the basic systems are investigated in the vicinity of a given program manifold. Nonlinearities satisfy to the conditions of local quadratic connection. Sufficient conditions for the absolute stability of the program manifold with respect to a given vector function are obtained by constructing the Lyapunov function, “quadratic form plus integral of nonlinearity”. A specific method for the selection of the Lyapunov matrix is ​​indicated.

Consider the problem of constructing, for a given program manifold , an automatic control system of the following structure [3]

, (4.1.1)

where  is some n-vector function satisfying the conditions for the existence of a solution; are matrices; is -vector of control by deviation from the given program, satisfying the conditions

, (4.1.2)

. Here  is the class of continuous, continuously differentiable and norm bounded matrices. The given program  is exactly realized only if the initial values of the state vector satisfy the condition . However, this condition cannot be exactly satisfied, because of always there exist initial and permanent acting perturbations. Therefore, the conditions of the stability of the program manifold  with respect to the vector function  should be additionally required in the construction of systems of program motion.

Taking into account that the manifold is integral for system (4.1.1), we obtain

, (4.1.3)

where  and  - is the Erugin vector function [3] that satisfies the condition . If we assume that  , is a Hurwitz matrix, then differentiating the manifold with respect to time t by virtue of system (4.1.1) taking into account relations (4.1.3), we obtain

 (4.1.4)

 (4.1.5)

In the space  we choose the region  as follows

 (4.1.6)

Definition 4.1.1. A set  is called an integral manifold of equation (4.1.1 ) if, from that  follows  for all .

Definition 4.1.2. A program manifold  is called absolutely stable with respect to vector-function  if it is asymptotically stable in whole for solution of equations (4.1.4) for all  and the function  satisfying conditions (4.1.5).

Statement of the problem. To get the condition of absolute stability of a program manifold  of the non-autonomous basic control systems in relation to the given vector-function .

By using the generalized Lyapunov theorem [3, p. 226], we can now formulate the following theorem:

Main Theorem. If there exists a function continuously differentiable in domain (4.1.6), positive-definite, and admitting the upper limit in the whole, such that its derivative

 (4.1.7)

is a positive-definite function for any function satisfying conditions (4.1.5), then the program manifold is absolutely stable with respect to the vector function .

a) Asymptotic Stability of a Program Manifold of the Linear System. We first consider a linear nonautonomous system for the vector function 

 (4.1.8)

If for this system, we construct the Lyapunov function

, (4.1.9)

then the derivative of this function with respect to time t has the form

, (4.1.10)

where is the symmetric matrix

. (4.1.11)

Let the matrix be nonsingular and let the matrices and satisfy the matrix equality

, (4.1.12)

where is an arbitrary matrix. Then we can take the matrix  in the form

. (4.1.13)

By virtue of (4.1.13) and relations (4.1.11), we obtain

. (4.1.14)

By the Kronecker–Capelli theorem, the matrix satisfying (4.1.12) always exists.

Theorem 4.1.1 If the matrix of system (4.1.8) is nonsingular and, together with the matrix satisfies equality (4.1.12), then, for any given quadratic form with the matrix , there exists a unique quadratic form  with the matrix satisfying the equation

. (4.1.15)

Theorem 4.1.2 For the asymptotic stability in the whole of the program manifold of a linear system with variable coefficients with respect to the vector function , it is sufficient to demand the validity of the relations

. (4.1.16)

b) Absolute Stability of the Program Manifold of the Main Control System. We now consider system (4.1.4), (4.1.5). For this system, we construct a Lyapunov function of the form (4.1.9). Differentiating this function with respect to time t, by using system (4.1.4), (4.1.5) and applying the S-procedure, we obtain

, (4.1.17)

where , , , and S is determined from relation (4.1.5).

Denote . For to be positive-definite, it suffices that the condition be satisfied. By the main theorem, the following theorem is true:

Theorem 4.1.3 For the absolute stability of the program manifold of the main automatic control system with variable coefficients with respect to the vector function , it is sufficient that the relations , and the conditions (4.1.5) be true; here, the matrix is given by relation (4.1.13) and is determined from relation (4.1.12).

**4.2 The problem of stability of the program manifold of non-autonomous indirect control systems with stationary nonlinearities**

In a class of continuously-differentiable at times t and bounded on a norm matrices  we consider the program manifold , which is integral for the system

  (4.2.1)

provided , where is a state vector of the object, is a vector-function, satisfying to conditions of existence of a solution , are continuous matrices, is a vector, is a vector-function of control on deviation from given program manifold, satisfying to conditions of local quadratic connection

. (4.2.2)

Due to the fact that is the integral manifold for the system (4.2.1) and (4.2.2) we have

 (4.2.3)

here is the Jacobi matrix and is a certain s-dimensional Erugin vector function, satisfying conditions  [3].

When solving inverse problems of the dynamics of automatic control systems, the main and obligatory requirement is the stability of program motion in the presence of unstable acting elements and systems that deviate from the given programs with the initial data.

Taking into account that is the integral manifold for the system (4.2.1), and by choosing the Erugin function as following

 (4.2.4)

here is Hurwitz matrix and differentiating the manifold Ω(t) with respect to time t along the solutions of system (4.2.1), we get [1]

.  (4.2.5)

Here nonlinearity satisfies to conditions (4.2.2).

Definition 4.2.1 A program manifold Ω(t) is called absolutely stable, if it is asymptotically stable on the whole at all functions satisfying to the conditions (4.2.2).

Statement of the problem. To get the conditions of absolute stability of a program manifold Ω(t) of the indirect control systems with variable coefficients in relation to the given vector-function ω.

Some survey on the construction of control systems with variable coefficients was given in [38].

First, we consider the following system with variable coefficients as a linear approximation of the system (4.2.5), (4.2.2) with respect to the vector function 

, . (4.2.6)

We construct a Lyapunov function for the system (4.2.6) in the form .

Theorem 4.2.1 Let the Erugin function have the form (4..2.4). Then, if the matrix  is non-degenerate and together with the matrix  satisfy to equality , then whatever the given quadratic form with the matrix there exists a unique quadratic form with the matrix and satisfies the equation .

Theorem 4.2.2. Let the Erugin function  has the form (4..2.4). Then for asymptotic stability in the whole of the program manifold of a linear system with variable coefficients relative to the vector function  it is sufficient fulfillment of relations , .

The basic theorem. If there is a real, continuous differentiable function of in the given domain and positive-definite and allowing the highest limit in whole such that its derivative would be definitely positive for any function satisfying conditions (4.2.2), then the program manifold is absolutely stable with respect to vector functions .

**4.3 Stability of the program manifold of control systems with non-stationary nonlinearity**

a) We introduce into consideration a class of continuously-differentiable at times t and bounded on a norm matrices .

Let the program manifold as follows is given, which is integral for the system

 (4.3.1)

where  is a state vector of the object, is a vector-function, is a vector-function of control on deviation from given program manifold, satisfying to conditions of local quadratic connection

 (4.3.2)

, (4.3.3)



Note that the following estimate

, (4.3.4)

can be obtained from the condition (4.3.2), where are the smallest, largest eigenvalues of matrices .

Due to the fact that  is the integral manifold for the system (4.3.1), taking into account that ,  is the Erugin vector function, satisfying conditions  [1] and by choosing F as the following

 (4.3.5)

we get the following system with respect to the vector-function ω:

 (4.3.6)

Statement of the problem. To get the condition of absolute stability of a program manifold Ω(t) of the non-autonomous basic control systems with non-stationary nonlinearity with respect to the vector-function ω.

For the system (4.3.6) we construct a Lyapunov function of the form

 (4.3.7)

Taking into account the properties (4.3.2) and making the substitution we obtain an estimate

, (4.3.8)

Here  are the smallest and largest eigenvalues of matrices ;

On the basis of property (4.3.4) and the substitution a derivative of the function (4.3.7) takes the form

, (4.3.9)

where  expressed through the date of system (4.3.6).

Due to the fact that  the following estimates hold

 (4.3.10)

where   are the smallest and largest eigenvalues of matrix . Taking into account the estimates (4.3.4) from (4.3.10), we get

 (4.3.11)

Theorem 4.3.1 Let the Erugin function have the form (4.3.5). Suppose also that there are matrices such that  and the nonlinear vector-function satisfies conditions (4.3.2) - (4.3.4). Then, for the absolute stability of the program manifold  of system (4.2.5) with respect to a given vector function , it is sufficient to satisfy relations (4.3.8) and (4.3.11).

b) In a class of matrices  we consider the program manifold , which is integral for the system

 (4.3.12)

provided .

Here  a nonlinear vector-function  satisfies to conditions

 (4.3.13)

According to our assumption we have

, (4.3.14)

where the Erugin function [3]: , . Let the system (4.3.12) has the property of asymptotic stability.

Statement of the problem. To get the condition of absolute stability of a program manifold Ω(t) of the non-autonomous indirect control systems with non-stationary nonlinearity (24), (25) with respect to the given vector-function ω.

For the system (4.3.14) we construct a Lyapunov function of the form

 (4.3.15)

Based on property (4.3.13), the function (4.3.15) satisfies the estimates

 (4.3.16)

Here are real, positive, continuous, smallest and largest roots of the characteristic equation   Differentiating the Lyapunov function (4.3.15) by virtue of system (4.3.14) in time t, we have

 (4.3.17)

where are expressed through the data of the system (4.3.14). Based on inequality (4.3.17) we have

 (4.3.18)

Here  are continuous, real, positive, smallest and largest roots of the characteristic equation .

Theorem 4.3.2 Let the Erugin function  have the form (4.3.5). Suppose also that there are matrices such that and the nonlinear vector-function  satisfies conditions (4.3.13). Then, for the absolute stability of the program manifold  of the indirect control system (4.3.14) with respect to a given vector function , it is sufficient to satisfy relations (4.3.16) and (4.3.18).

**5 Investigation of the stability of difference-dynamic systems**

**5.1 Poisson stability of difference-dynamical systems**

The Poisson stability of difference dynamic systems is investigated using the discrete shift operator [39]. Let us consider the following difference dynamic system

(5.1.1)

here is vector function, definite and continuous together with its partial derivatives on a direct product

The fulfillment of the indicated conditions means that the conditions of the following existence and uniqueness theorem are satisfied for the system (5.1.1).

Theorem 5.1.1 Let

(5.1.2)

be some point of the set Then for all point (5.1.2) there is a solution ξ (n) of the difference-dynamic system (5.1.1) with the initial condition

(5.1.3)

defined on some interval containing the point Moreover, if there are two solutions with the same initial condition (5.1.3), each of which is defined on its set containing the point , then these solutions coincide in the common area of their definition.

Let

(5.1.4)

be solution of the difference-dynamic system (5.1.1), defined on Let

(5.1.5)

solution of the same system, but defined on some other interval . We will say that solution (5.1.5) is a continuation of the solution (5.1.4), if interval contains interval Solution (5.1.5) coincides with solution (5.1.4) on the interval In particular, it is considered that solution (5.1.5) is a continuation of solution (5.1.4) if the interval contains interval And solution (5.1.5) coincides with the solution (5.1.4) on Solution (5.1.5) is a continuation of solution (5.1.4) even in the case when the intervals and coincide, as solutions (5.1.4) and (5.1.5) completely coincide.

Definition 5.1.1 A point is called Poisson positively stable if for each neighborhood and for each positive number one can specify such number that Similarly, a point is called Poisson negatively stable if for each neighborhoods and for every positive number one can specify such number such that And, finally, the Poisson stable point, both positively and negatively, is called Poisson stable [40-42].

Theorem 5.1.2 If a point is positively Poisson stable, then each point of the trajectory, described by the motion is also positively Poisson stable. A similar statement holds for points that are negatively Poisson stable.

**5.2 Lagrange stability of the difference dynamic systems**

Using a discrete analogue of the second Lyapunov method, the necessary and sufficient Lagrange stability of the difference dynamic systems was obtained.

Let us consider the difference-dynamic system

 (5.2.1)

Let  be solution with initial condition  It's clear that

1. This solution can be continued for all . Then the solution  can be extended indefinitely.
2. There is such that for . Then the solution  has a finite definition time.
3. The solution  is bounded.

The two possibilities a) and b) are clearly incompatible, but they complement each other. The third case c) is compatible with a) and it is not compatible with b).

Definition 5.2.1 Difference-dynamic system (5.2.1) is called Lagrange stable if

1) solutions  for , here ;

2)  is bounded on .

For example, if an difference-dynamic system (5.2.1) has a bounded solution that is asymptotically stable on the whole, then this difference-dynamic system is Lagrange stable.

Theorem 5.2.1 Difference-dynamic systems (5.2.1) is Lagrange stable if and only if there exists the function  on  such that

1) here ;

2) the function  is not increasing for all solution 

**5.3 On the dissipativity of dynamic difference systems**

In practice, where dynamic difference systems are used, dynamic difference systems are very often encountered, in which, due to natural dissipation, each solution after a sufficiently long moment of time falls into a certain fixed region and remains in it in the future. Such dynamic difference systems are called dissipative dynamic difference systems [43].

The main objectives of the study of dynamic difference systems for dissipativity are the problem of obtaining a criterion for belonging of the considered dynamic difference systems to the class of dissipative systems or similar classes of systems [44].

Let us consider the nonlinear difference-dynamic system

 (5.3.1)

here  is set of natural numbers,  and  - - dimensional vector column, .

Definition 5.3.1 The difference-dynamic system (5.3.1) is called dissipative system or -system if there exists such that

 (5.3.2)

here  is solution of the (5.3.1) with initial data , .

It follows from the definition itself that all solutions of the dissipative difference-dynamic system are extended to all moments .

Theorem 5.3.1 The difference-dynamic system (5.3.1) belongs to dissipative classes if there exists a Lyapunov-type function  that in the domain

 (5.3.3)

has the following properties:



If  then  (5.1) is limited solution.

Theorem 5.3.2 Let ∃ in and



Then (5.3.1) belongs to the class of dissipative difference-dynamic systems.

**5.4 On the total stability theorem for the difference-dynamic systems**

Let the difference-dynamic system

 (5.4.1)

be given. Here the right-hand side is defined in some real region for all 

Arbitrary functions , that do not vanish at all, are called perturbations, which satisfy the condition in the domain . Here  is a sufficiently small positive number.

Let

 (5.4.2)

be some solution of (5.4.1). Let

 (5.4.2/)

be some solution of the following difference-dynamic system without perturbation

 (5.4.1/)

We will compare the solution (5.4.2/) of the (5.4.1/) and the solution (5.4.2) of the (5.4.1). It is required to find out whether it is possible, for any predetermined arbitrarily small number and any predetermined value , to find such arbitrarily small other two numbers and depending in general on  and , that if the conditions are satisfied, the inequality  takes place for all finite values .

If it takes place, then we say that the motion (state) of the difference-dynamic system, which is determined by the solution (5.4.2/) of the (5.4.1/) is totally stable, otherwise it is totally unstable with respect to the same perturbations.

If the state, determined by the solution (5.4.2/) of the (5.4.1/), is stable with respect to perturbations and, if, in addition, the condition  is satisfied, then we say that the state determined by the solution (5.4.2/) of the (5.4.1/) is totally asymptotically stable.

If there  and are functions of only and they do not depend on , then we say that the state determined by the solution (5.4.2/) of the (5.4.1/) is totally uniformly stable.

The main theorem. Consider the following difference-dynamic system

, (5.4.3)

on . Here  and , are defined on  Also consider the unperturbed difference-dynamic system

. (5.4.4)

Suppose that equation (5.4.4) has a solution  which is defined for all and , and together with its some -neighborhood  remains inside the set .

Theorem 5.4.1 Let the function  is limited on  and satisfies the Lipschitz condition , .

Let the function  be uniformly continuous in  with respect to and bounded on . Let the solution  of the (5.4.4) be uniformly asymptotically stable.

Then for any , there exist  and  such that for all solutions  of the (5.4.3) with initial values satisfying the inequality , the inequality  holds.

**5.5 Investigation of the Poincaré mapping near a homoclinic loop**

In a neighborhood of a fixed point at the origin, the Poincaré mappping can be written in the following form

 (5.5.1)

Here  is a nondegenerate dimensional matrix, . Before considering the nonlinear mapping (5.5.1), it is useful to study the behavior of the trajectories of the linearized mapping

 (5.5.2)

The difference between (5.5.2) and (5.5.1) is that the solving of the (5.5.1) is essentially an impossible problem. In this connection, a natural question arises, which was first formulated by Poincaré for differential equations. Under what conditions the trajectories (5.5.1) near the equilibrium state behave similarly to the trajectories of the linearized mapping (5.5.2).

In modern terminology, the behavior of two systems is called analogous if the systems are topologically equivalent.

The problem of topological classification of rough equilibrium states of a mapping is solved in the following theorem.

Theorem 5.5.1 (Grobman – Hartman analogy for mappings). Let point *O* be a rough state of equilibrium. Then there are neighborhoods and in which the original mapping and linearized mapping are equivalent.

In this case, the equilibrium state of a given nonlinear Poincaré mappping is said to be locally topologically equivalent to the equilibrium state of its linear part (5.5.1).

Theorem 5.5.2 A formal change of variables reduces the mapping (5.5.1) to . Here  is a formal series.

Theorem 5.5.3 If the eigenvalues of the matrix  are nonresonant and they belong to the Poincaré domain, then the mapping (5.5.1) with an analytic right-hand side can be brought to a linear form by an analytic change of variables.

Poincaré proved this theorem for differential equations with the help of morant series.

**CONCLUSION**

This report contains investigations on theme «Development of methods for solving the inverse problems of stochastic differential systems and their application» carried out in 2018-2020 in the field of qualitative theory of the ordinary differential equations. The obtained results are aimed at investigation of the solvability of inverse problems of the differential systems in the presence of random perturbations, investigation of the influence of random perturbing forces on solvability of inverse problems of dynamics and stability of the given properties of motion.

The inverse problem of stochastic differential systems was solved by the method of Liouville and of Shulgin additional variables. The inverse problem for stochastic Helmholtz systems was solved.

The inverse problem for Birkhoff stochastic systems was solved. The force field was constructed along the given trajectories in the presence of random perturbations.

The set of comparison’s vector functions of the program motion that are independent on time and dependent on time was constructed in the presence of random perturbations. The general problem of constructing the set of stochastic equations of program motion and the set of comparison was solved.

Sufficient conditions for the stability of nonautonomous systems’ program manifold of direct control and of indirect control with stationary and nonstationary nonlinearities was obtained.

The research on stability of the difference dynamical systems by Poisson and by Lagrange was investigated. The research on the Poincaré map near the homoclinic loop was carried out.

This report presents investigations covering the theory of inverse problems of stochastic differential systems and Lyapunov stability theory which have been widely used in recent years in the study of complex nonlinear processes. The obtained results can be used in specific investigations in related sciences. The high level of scientific researches is characterized by the participation of performers of the theme in a number of international conferences, also publications in authoritative mathematical journals which are listed in Appendix A (the list of published papers of this report).

**LIST OF USED SOURCES**

1. Galiullin A.S. Methods for solving inverse problems of dynamics. - Moscow: Nauka, 1986. - 224 p. (in Russian)
2. Mukharlyamov R.G. On the construction of systems of differential equations of motion of mechanical systems // Differential equations. - Moscow, 2003. - Vol. 39, No. 3. - P. 343-353. (in Russian)
3. Yerugin N.P. Construction of the entire set of systems of differential equations having a given integral curve // ​​Applied Mathematics and Mechanics. - Moscow, 1952. - Vol.10, vol. 6. - P. 659-670. (in Russian)
4. Galiullin A.S. To the problem of constructing systems of program motion // Automation and telemechanics. - 1970. - No. 3. - C.32-37. (in Russian)
5. Galiullin A.S. To the problem of constructing systems of differential equations // Differential equations. - Moscow, 1970. - Vol.6, No. 8. - P.1343-1348. (in Russian)
6. Mukharlyamov R.G., Kirgizbaev Zh.K. Control of program motion and inverse problems of dynamics of systems with variable mass. - Shymkent: Nurly Beyne, 2008. - 180 p. (in Russian)
7. Mukhametzyanov I.A., Mukharlyamov R.G. Equations of program motions. - Moscow: Publishing House of the Peoples' Friendship University, 1986. - 88 p. (in Russian)
8. Mukharlyamov R.G., Abramov N.V. Control of motion along a given curve and inverse problems of dynamics // Bulletin of the Russian University of Peoples' Friendship. A series of mathematics, computer science, physics. - 2011. - No. 2. - P. 104-110. (in Russian)
9. Karachanskaya EV The construction of program controls for a dynamical system based on the set of its first integrals // Contemporary Mathematics. Fundamental directions. - Moscow, 2011. - Vol. 42. - P. 125-133. (in Russian)
10. Budochkina S.A. On the representation of a single operator equation with the first time derivative in the form of a - Hamiltonian equation // Differential equations. - Moscow, 2013. - Vol. 49, No. 2. - P. 175-185. (in Russian)
11. Galiullin A.S. Helmholtz systems. - Moscow: Nauka, 1995. - 86 p. (in Russian)
12. Galiullin A.S. Selected works in two volumes. - Moscow: RUDN, 2009. - Vol. I, II. – 462 p. (in Russian)
13. Helmholtz G. On physical meaning of the principle of least action // Variational principles of mechanics. - Moscow, 1959. - P. 430-459. (in Russian)
14. Suslov G.K. On Helmholtz's kinetic potential // Matematicheskii sbornik. - 1896. - Vol.19, No. 1. - P. 197-210. (in Russian)
15. Mayer A. Die existenzbeingungen eines kinetischen potentiales // Ber. Verhand. Kgl. Sachs. Ges. Wiss. Leipzig. - 1896. - Vol. 48. - P. 519-529.
16. Santilli R.M. Foundations of Theoretical Mechanics. The Inverse Problem in Newtonian Mechanics. - New York: Springer-Verlag, 1978. - 266 p.
17. Filippov V.M., Savchin V.M., Shorokhov S.G. Variational principles for non-potential operators // Itogi Nauki i Tekhniki. Modern problems of mathematics. Recent achievements / VINITI. - Moscow, 1992. - Vol. 40. - P. 3-178. (in Russian)
18. Galiullin A.S. Stability of motion. - Moscow: Nauka, 1973. - 104 p. (in Russian)
19. Mukhametzyanov I.A. On the stability of program manifold // Differential equations. - Moscow, 1973. - Vol.9, No. 5. - P. 846-856. (in Russian)
20. Mukharlyamov R.G. On the construction of the systems’set of differential equations of stable motion with respect to an integral manifold // Differential equations. - Moscow, 1969. - Vol.5, No. 4. - P. 688-699. (in Russian)
21. Tleubergenov M.I. Necessary and sufficient conditions for the stability of an integral manifold // Differential equations and inverse problems of dynamics. - Moscow: Izd-vo UDN, 1983. - P. 125-132. (in Russian)
22. Sokolov A.V. On conditions of motion’s asymptotic stability of three-link electromechanical manipulator // Problems of mechanics and control: Nonlinear dynamical systems. - 2009. - No. 41. - P. 156-172. (in Russian)
23. Mukharlyamov R.G., Matukhina O.V. Modeling of control processes, stability and stabilization // Bulletin of Kazan Technological University. - 2012. - Vol. 15, No 12. -P. 220-224. (in Russian)
24. Azimov D.M., Mukharlyamov R.G. Analytic synthesis of extreme trajectories and stability of program motion // Bulletin of the Russian University of Peoples' Friendship. A series of mathematics, computer science, physics. - 2012. - No 4. - P. 87-95. (in Russian)
25. Khasminsky R.Z. Stability of systems of differential equations for random perturbations of their parameters. - Moscow: Nauka, 1969. - 368 p. (in Russian)
26. Sagirov P. Stochastic methods in the dynamics of satellites // Mechanics. Period. Sat. translations of foreign. articles. - Moscow, 1974. - No. 5 (147). - P.28-47. (in Russian)
27. Sinitsyn I.N. On the fluctuations of a gyroscope in a cardan suspension // Izvestiya AN SSSR. Mechanics of a solid body. - Moscow, 1976. - No. 3. - P. 23-31. (in Russian)
28. Gikhman II, Skorokhod A.V. Stochastic differential equations. - Kiev: Naukova Dumka, 1968. - 356 p. (in Russian)
29. Watanabe S., Ikeda N. Stochastic differential equations and diffusion processes. - Moscow: Nauka, 1986. - 445 p. (in Russian)
30. Whittaker E.T. Analytical dynamics. - Moscow-Leningrad, 1937. - 500 p. (in Russian)
31. Shulgin M.F. On some differential equations of analytical dynamics and their integration. - Tashkent, 1958. - 183 p. (in Russian)
32. Santilli R.M. Foundation of Theoretical Mechanics. 2. Birkhoffian Generalization of Hamiltonian Mechanics. – New-York, Springer-Verlag, 1983. – 370 p.
33. Pugachev V.S., Sinitsyn I.N. Stochastic differential systems. Analysis and filtering. - Moscow: Nauka, 1990. - 632 p. (in Russian)
34. Sonin N.Ya. On the determination of the maximum and minimum properties of plane curves // Bulletin of the Warsaw University. – 1886. - No.1. – P. 1-68. (in Russian)
35. Darboux G. Leçons sur la théorie générale des surfaces. V.3. – Paris, Gautier-Villars, 1894. – 512 p.
36. Rush N., Abets P., Lalua M. Lyapunov's direct method in stability theory. - Moscow: Mir, 1980. - 300 p. (in Russian)
37. Vassilina G.K., Tleubergenov M.I. Solution of the Problem of Stochastic Stability of an Integral Manifold by the Second Liapunov Method // Ukrainian Mathematical Journal. – 2016. –Vol. 68, №1. – P. 14-28.
38. Zhumatov S.S. Absolute stability of a program manifold of non-autonomous basic control systems // News NAS RK. Phys.-Math. Ser. – 2018. – №. 6. – P. 37-43.
39. Krasnoselsky M.A. Shift operator along trajectories of differential equations. - Moscow: Nauka, 1966. - 331 p. (in Russian)
40. Krasnoselsky M.A., Zabreiko P.A. Biometric methods of nonlinear analysis. - Moscow: Nauka, 1975. - 512 p. (in Russian)
41. Nemytskiy V.V., Stepanov V.V. Qualitative theory of differential equations. - Moscow: Gittl, 1949. - 550 p. (in Russian)
42. Sibirskiy K.S. Introduction to Topological Dynamics. - Kishinev, 1970. - 144 p. (in Russian)
43. Demidovich B.P. On the dissipativity of some nonlinear system of differential equations // Vestnik MGU. - 1961. - No.6. - P. 18-27. (in Russian)
44. Aleksandrov A.Yu., Zhabko A.P. Stability of difference systems. Tutorial. - St. Petersburg, 2003. - 111 p. (in Russian)

**APPENDIX A**

**List of publications**

1 Vasilina G. K., Tleubergenov M.I. On the optimal stabilization of an integral manifold // Journal of Mathematical Sciences. – 2018. – Vol. 229, No. 4. – Pp. 390-402. http://DOI 10.1007/s10958-018-3684-5. (Scopus SJR=0.284, Процентиль 12 в категории General Mathematics).

2 Zhumatov S.S. Absolute stability of the programmatic variety of indirect control systems with discontinuous nonlinearities // Bulletin of the International Kazakh-Turkish University. HA. Yasawi. Series of mathematicians, physics, informatics. Special issue based on the materials of the conference of mathematicians of Kazakhstan "Actual problems of mathematics". - 2018. - Vol. I, No. 1 (4). - P. 46-50. (in Russian)

3 Tleubergenov M.I., Ibraeva G.T. On the solvability of the inverse problem of closure of stochastic differential systems // Bulletin of the International Kazakh-Turkish University. HA. Yasawi. Series of mathematicians, physics, informatics. Special issue based on the materials of the conference of mathematicians of Kazakhstan "Actual problems of mathematics". - 2018. - Vol. I, No. 1 (4). - P. 140-144. (in Russian)

4 Bapaev K.B., Vasilina G.K., Slamzhanova S.S. On the existence of m-parametric summable manifolds for difference-dynamical systems // Mathematical journal. - 2018. - Vol. 18, No. 2 (68). - P. 19-30. (in Russian)

5 Azhymbaev D., Tleubergenov M. On the construction of a strength function in a probabilistic case // Differential operators and modeling of complex systems: abstracts of the traditional international scientific April conference IMMM. - Almaty, 2018. - P. 37-38. (in Russian)

6 Zhumatov S. Stability of a program manifold of nonautomous basic control systems // Differential operators and modeling of complex systems: abstracts of the traditional international scientific April conference IMMM. - Almaty, 2018. - P. 34-35.

1. Bapaev K., Slamzhanova S. On the existence of summable manifolds for difference dynamical systems // Differential operators and modeling of complex systems: abstracts of the traditional international scientific April conference IMMM. - Almaty, 2018. - P. 41-42. (in Russian)
2. Tleubergenov M.I., Azhymbaev D.T. On the solvability of the inverse stochastic problem of constructing the strength function // Proceedings of the international scientific conference on differential equations "Erugin readings-2018" - Minsk, 2018. - P. 108-109. (in Russian)
3. Zhumatov S.S. Stability of a program manifold of basic control systems // Proceedings of the international scientific conference on differential equations "Erugin readings-2018" - Minsk, 2018. - P. 139-140.
4. Tleubergenov M., Azhymbaev D. On stochastic inverse problem of construction of the force function // Abstracs of the International Scientific Conference “Mathematical Analysis, Differential Equations & Applications, MADEA 8. – Cholpon-Ata, 2018. – P. 126-127.
5. S. Zhumatov. Stability of a program manifold of non-autonomous basic control systems // Abstracs of the International Scientific Conference “Mathematical Analysis, Differential Equations & Applications, MADEA 8. – Cholpon-Ata, 2018. – P. 130-131.
6. Vassilina G., Tleubergenov M. On construction of the set of comparison functions of the program motion in the probable statement // Fourth International Conference on Analysis and Applied Mathematics: the abstract book of the conference ICAAM 2018. – Turkey, 2018. – P. 177.
7. Tleubergenov M.I., Ibraeva G.T. On solving of the stochastic Helmholtz problem by the method of additional variables of Liouville // Modern problems of mathematics and its application in natural sciences and information technologies: Proceedings of the international scientific conference dedicated to the 50th anniversary of the Faculty of Mathematics and Informatics of the Y. Fedkovych Chernovetsk National University. - Chernivtsi, 2018. - P. 40.
8. Zumatov S. Stability of a program manifold of non-autonomous control systems // Modern problems of mathematics and its application in natural sciences and information technologies: Proceedings of the international scientific conference dedicated to the 50th anniversary of the Faculty of Mathematics and Informatics of the Y. Fedkovych Chernovetsk National University. - Chernivtsi, 2018. - P. 42.
9. Vasilina G.K., Tleubergenov M.I. On the construction of stochastic differential equations and comparison functions for a given integral manifold // Differential equations and related problems: Proceedings of an international scientific conference. - Sterlitamak, 2018. - P. 318-319. (in Russian)
10. Zhumatov S.S. Absolute stability of the program manifold of the basic control theorem with discontinuous nonlinearities // Informatics and Applied Mathematics: Proceedings of the III International Scientific Conference. Part 1. - Almaty, 2018. - P. 130-138. (in Russian)
11. Tleubergenov M.I., Vasilina G.K. On the construction of a set of comparison functions for program motion in a probabilistic case // Informatics and Applied Mathematics: Proceedings of the III International Scientific Conference. Part 1. - Almaty, 2018. - P. 164-169. (in Russian)
12. Tleubergenov M.I. and Ibraeva G.T. On the Solvability of the Main Inverse Problem for Stochastic Differential Systems // Ukrainian Mathematical Journal. – 2019. –Vol. 71, No. 1. Р. 157-165. (Web of Science, IF – 0.345, Q4).
13. Tleubergenov M.I., Azhymbaev D.T. Stochastical problem of Helmholtz for Birkhoff system // Bulletin of the Karaganda University. Mathematics series. – 2019. – № 1(93). – P. 78-87 (Web of Science).
14. Tleubergenov M.I., Azhymbaev D.T. On the solvability of the stochastic Helmholtz problem // Nonlinear oscillations. - 2019. - Vol. 22, No. 3. - P. 398-405. (in Russian)
15. Tleubergenov M.I., Ibraeva G.T. On Inverse Problem of Closure of Differential Systems with Degenerate Diffusion // Eurasian Mathematical Journal. – Astana, 2019. – Vol. 10, № 2. – P. 146-155. (Web of Science, Scopus, SJR – 0.624).
16. Zhumatov S.S. On the stability of the program manifold of control systems with variable coefficients // Ukrainian Mathematical Journal. - 2019. - No. 8. - P. 1053-1063. (in Russian)
17. Zhumatov S.S. Stability of a program manifold of indirect control systems with variable coefficients // Mathematical Journal. – Almaty, 2019. –Vol. 19, No 2. –P. 121-130.
18. Bapaev K.B., Vasilina G.K., Slamzhanova S.S., Toleuova B.Zh. On the strong stability of difference-dynamic systems in the critical case under parametric perturbations and bifurcations // Bulletin of the Almaty University of Energy and Communication. –2019. - No. 2 (45). - P 94-101. (in Russian)
19. Vasilina G., Tleubergenov M. On the construction of a set of stochastic differential equations of stable program motion // Traditional April International Mathematical Conference in honor of the Day of Science Workers of the Republic of Kazakhstan and Workshop "Problems of modeling processes in electrical contacts": abstracts. - Almaty, 2019. - P. 114-115. (in Russian)
20. Bapaev K., Bapaeva S., Slamzhanova S. On the stability of difference-dynamic systems with lagging argument // Traditional April International Mathematical Conference in honor of the Day of Science Workers of the Republic of Kazakhstan and Workshop "Problems of modeling processes in electrical contacts": abstracts. - Almaty, 2019. - P. 51-52. (in Russian)
21. Zhumatov S. Absolute stability of a program manifold of non-autonomous indirect control systems with stationary nonlinearities // Traditional April International Mathematical Conference in honor of the Day of Science Workers of the Republic of Kazakhstan and Workshop "Problems of modeling processes in electrical contacts": abstracts. - Almaty, 2019. - P. 106-107.
22. Zhumatov S.S. Absolute stability of a program manifold of control systems with variable coefficients // Proceedings of the International Scientific Conference "Theoretical and Applied Problems of Mathematics, Mechanics and Informatics". - Karaganda, 2019. - P. 56-57. (in Russian)
23. Tleubergenov M.I., Azhymbaev D.T.On the method of additional variables in the stochastic Helmholtz problem // Proceedings of the International Scientific Conference "Theoretical and Applied Problems of Mathematics, Mechanics and Informatics". - Karaganda, 2019. - P. 102-103. (in Russian)
24. Vassilina G., Tleubergenov M. On construction of the comparison function of program manifold of the stochastic differential equations // III International conference on mathematics «An Istanbul meeting for world mathematicians»: abstracts book. – Istanbul, 2019. – P. 162.
25. Zhumatov S.S. On the stability of a program manifold of control systems with variable coefficients // Ukrainian Mathematical Journal. – 2020. –Vol. 71, No 8. – pp. 1202-1213 (Web of Science, IF – 0.345, Q4). DOI: <https://doi.org/10.1007/s11253-019-01707-7>.
26. Bapaev K.B., Vassilina G.K., Slamzhanova S.S. On compressibility area of unstable difference-dynamic systems and determinated chaos // News of the National Academy of Sciences of the Republic of Kazakhstan. Physico-mathematical series. – 2020. – № 4. – P. 103-113. (Web of Science).
27. Vasilina G.K., Tleubergenov M.I. On the inverse problem of constructing stochastic differential equations and comparison functions for given properties of motion // Abstracts of the Traditional International April Mathematical Conference in honor of the Day of Science Workers of the Republic of Kazakhstan. - Almaty, 2020. - P. 157-158. (in Russian)
28. Bapaev K.B., Slamzhanova S.S. Investigation of stability in the critical case of m-pairs of complex conjugate multipliers of the Poincaré map // Abstracts of the Traditional International April Mathematical Conference in honor of the Day of Science Workers of the Republic of Kazakhstan. - Almaty, 2020. - P.155-156. (in Russian)
29. Zhumatov S. Absolute stability of a program manifold of non-autonomous control systems with non-stationary nonlinearities // Abstracts of the Traditional International April Mathematical Conference in honor of the Day of Science Workers of the Republic of Kazakhstan. - Almaty, 2020. - P. 144-145.
30. Tleubergenov M.I., Ibraeva G.T. On the closure of stochastic differential equations of motion // Eurasian mathematical journal. – 2020. (A revised version of the article was sent taking into account the comments and wishes of the reviewer in February 2020. A positive review was received in March 2020. In the press. Volume: 6 pages.) (Web of Science, Scopus, SJR – 0.624).
31. Tleubergenov, M.I., Azhymbaev, D.T., On Solvability of Helmholtz Stochastic Problem // Journal of Mathematical Sciences (In the press, November-December 2020, volume: 8 pages.) (Scopus SJR=0.284, Процентиль 12 в категории General Mathematics).
32. Stanzhitskii A.N., Karakenova S.G., Zhumatov S.S. On a comparison theorem for stochastic integro-functional equations of neutral type // Journal of Mathematics, Mechanics, Computer Science. – 2020. – No 1(105). – P. 30-45.
33. Zhumatov S.S. On the absolute stability of a program manifold of non-autonomous control systems with non-stationary nonlinearities // Kazakh Mathematical Journal. – Almaty, 2020. –Vol. 20, No 4. (n the press, volume: 9 pages.)

**APPENDIX B**

**Work schedule**









**APPENDIX B**

**Work schedule**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Phase refe  rence code | The name of works under the contract and the main stages of its implementation | Deadline | | | Expected Result | |
| Start | | Ending |
| I | To solve the inverse  problem of stochastic  differential systems using the method of Liouville  additional variables. | January 2018 | | March  2018 | The inverse problem of  stochastic differential systems will be solved by the method of Liouville additional  variables. | |
| II | To solve the inverse  problem for stochastic Birkhoff systems. | April  2018 | | June  2018 | The inverse problem for Birkhoff stochastic  systems will be solved. | |
| III | To construct the set of  comparison’s vector  functions of the program  motion that are independent of time in the presence of random perturbations. | July  2018 | | September  2018 | The set of comparison’s vector functions of the program motion that are independent of time will be constructed in the presence of random  perturbations. | |
| IV | To solve the problem of  stability of nonautonomous systems’ program manifold of direct control with  stationary nonlinearities. | October  2018 | | 1 November 2018 | The problem of  stability of  nonautonomous systems’ program manifold of  direct control with  stationary nonlinearities will be solved.  Sufficient conditions for the stability of  nonautonomous systems’ program manifold of  direct control with  stationary nonlinearities will be obtained. | |
| V | To research on stability of the difference dynamical systems by Poisson and by Lagrange. | October  2018 | | 1 November  2018 | To research on stability of the difference  dynamical systems by Poisson and by  Lagrange. | |
| VI | To solve the inverse problem of stochastic differential systems by the method of Shulgin additional variables. | January 2019 | March  2019 | | The inverse problem of stochastic differential systems will be solved by the method of Shulgin additional variables. |
| VII | To construct the force field along the given trajectories in the presence of random  perturbations. | April  2019 | June  2019 | | The force field will be constructed along the  given trajectories in the presence of random  perturbations. |
| VIII | To construct the set of  comparison’s vector functions of the program motion that are dependent of time in the  presence of random  perturbations. | July  2019 | September  2019 | | The set of comparison’s vector functions of the program motion that are dependent of time will be constructed in the  presence of random  perturbations. |
| IX | To solve the problem of  stability of nonautonomous systems’ program manifold of indirect control with stationary nonlinearities. | October  2019 | 1 November 2019 | | The problem of  stability of  nonautonomous systems’ program manifold of  indirect control with  stationary nonlinearities will be solved.  Sufficient conditions for the stability of  nonautonomous systems’ program manifold of  indirect control with  stationary nonlinearities will be obtained. |
| X | To research the stability of the difference dynamical systems by Poisson and by Lagrange. | October  2019 | 1 November  2019 | | The research on stability of the difference  dynamical systems by Poisson and by Lagrange will be carried out. |
| XI | To solve the inverse problem for stochastic Helmholtz  systems. | January 2020 | March  2020 | | The inverse problem for stochastic Helmholtz  systems will be solved. | | |
| XII | The force field will be  constructed along the given  trajectories in the presence of random perturbations. | April  2020 | June  2020 | | The force field will be constructed along the given trajectories in the presence of random  perturbations. | | |
| XIII | To solve the general problem of constructing a set of stochastic equations of program motion and a set of comparison  functions. | July  2020 | September  2020 | | The general problem of constructing a set of stochastic equations of program motion and a set of comparison  functions will be solved. | | |
| XIV | To solve the stability problem of program manifold of  control systems with  nonstationary nonlinearity. | October  2020 | 1 November 2020 | | The stability problem of program manifold of  control systems with  nonstationary  nonlinearity will be solved.  Sufficient conditions for the stability of program manifold of control  systems with  nonstationary  nonlinearity will be  obtained. | | |
| XV | To carry out the research on the Poincaré map near the  homoclinic loop. | October  2020 | 1 November  2020 | | The research on the Poincaré map near the homoclinic loop will be carried out.  Presumably, the results of scientific research carried out within the framework of the project will be formalized in the form of articles and sent to such journals as "Differential Equations", "Siberian Mathematical Journal", "Ukrainian Mathematical Journal", "Izvestiya Vuzov. Series of Mathematics", "Izvestiya Vuzov. Series of applied nonlinear dynamics "," Bulletin of RUDN. Series of mathematics, computer science, physics ","Open Engineering" or others.  As a result of the implementation of this project for the entire period, at least 3 articles will be published in peer-reviewed foreign scientific journals, indexed in the databases of Web Sciences or Scopus with a non-zero impact factor, as well as at least 2 publications in peer-reviewed foreign and domestic scientific journals with a non-zero impact factor. | | |