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| Нead of research work,  Lead Researcher,  PhD, assoc. professor | 12.10.2021  signature, data | A.E.Imanchiyev |

Almaty 2021 **LIST OF EXECUTORS**

|  |  |  |  |
| --- | --- | --- | --- |
|  | Нead of research work,  Lead Researcher,  PhD, assoc. professor | 12.10.2021 | A.E.Imanchiyev (Introduction, sections 1.1-1.3, 2.1-2.3, conclusion) |
|  | Executors: | | |
|  | Chief Researcher,  D-r of phys.-math. sci., professor | 12.10.2021 | A.T. Assanova  (sections 2.1-2.6) |
|  |  |  |  |
|  | Senior Researcher,  PhD | 12.10.2021 | S.Т. Mynbayeva  (sections 1.2-1.3) |
|  |  |  |  |
|  |  |  |  |
|  | Junior Researcher,  master | 12.10.2021 | A.A. Ermek  (section 1.1) |
|  | Norm controller,  PhD | 12.10.2021 | M.A. Sakhauyeva |
|  |  |  |  |

**ABSTRACT**

Report 45 pp., 1 book, 38 sources, 2 annexes

HIGH-ORDER DIFFERENTIAL EQUATIONS, PROBLEMS WITH NON-SEPARATED MULTIPOINT-INTEGRAL CONDITIONS, SOLVABILITY, PARAMETERIZATION METHOD, NEW GENERAL SOLUTION

The object of the research is problems with non-separated multipoint-integral conditions for high-order differential equations.

The goal of the research - Goal A: Construct algorithms of the parameterization method and establish solvability conditions for problems with non-separated multipoint-integral conditions for high-order differential equations; Goal B: Construct new general solutions of high-order differential equations, establish their properties and solvability conditions for problems with non-separated multipoint-integral conditions.

Dzhumabaev parameterization method, a modern methods of the theory differential equations and functional analysis are applied.

The following results are obtained:

A new general solution of high-order differential equations is constructed and its properties established. Using a new general solution, algorithms for finding solutions to the problem with non-separated multipoint-integral conditions for high-order differential equations are constructed. The conditions for the unique solvability of the problem with non-separated multipoint-integral conditions for high-order differential equations are established in terms of initial data. An effective method is developed for solving a problem with non-separated multipoint-integral conditions for high-order differential equations. A new general solution of families of high-order differential equations is constructed and its properties established. Algorithms for finding solutions to families of problems with non-separated multipoint-integral conditions for high-order differential equations are constructed, and the conditions for their unique solvability in terms of the initial data are established.

The research results are of theoretical importance and can be applied in the theory of multi-support beams, in problems of mathematical biology, as well as in mathematical modeling of problems for high-order differential equations.

**РЕФЕРАТ**

Есеп 45 бет, 1 кіт., 38 шығу к., 2 қос.

ЖОҒАРЫ РЕТТІ ДИФФЕРЕНЦИАЛДЫҚ ТЕҢДЕУЛЕР, БӨЛІНБЕГЕН КӨПНҮКТЕЛІ-ИНТЕГРАЛДЫҚ ШАРТТАРЫ БАР ЕСЕПТЕР, ШЕШІЛІМДІЛІК, ПАРАМЕТРЛЕУ ӘДІСІ, ЖАҢА ЖАЛПЫ ШЕШІМ

Зерттеу нысаны жоғары ретті дифференциалдық теңдеулер үшін үшін бөлінбеген көпнүктелі интегралдық шарттары бар есептер болып табылады.

Зерттеу мақсаты – А мақсат: жоғары ретті дифференциалдық теңдеулер үшін бөлінбеген көпнүктелі интегралдық шарттары бар есептердің параметрлеу әдісі алгоритмдерін құру және шешілімділігі шарттарын орнату; В мақсат: жоғары ретті дифференциалдық теңдеулердің жаңа жалпы шешімдерін тұрғызу, олардың қасиеттерін және бөлінбеген көпнүктелі интегралдық шарттары бар есептердің шешілімділік шарттарын орнату болып табылады.

Джумабаевтың параметрлеу әдісі, дифференциалдық теңдеулер мен функционалдық анализдің қазіргі әдістері қолданылған.

Келесі нәтижелер алынды:

Жоғары ретті дифференциалдық теңдеулердің жаңа жалпы шешімі құрылған және оның қасиеттері орнатылған. Жаңа жалпы шешім көмегімен жоғары ретті дифференциалдық теңдеулер үшін бөлінбеген көпнүктелі-интегралдық шарттары бар есептің шешімдерін табу алгоритмдері құрылған. Жоғары ретті дифференциалдық теңдеулер үшін бөлінбеген көпнүктелі-интегралдық шарттары бар есептің бірмәнді шешілімділігі шарттарын бастапқы берілімдер терминінде орнатылған. Жоғары ретті дифференциалдық теңдеулер үшін бөлінбеген көпнүктелі-интегралдық шарттары бар есепті шешудің тиімді әдісі әзірленген. Жоғары ретті дифференциалдық теңдеулер әулеттерінің жаңа жалпы шешімі құрылған және оның қасиеттері орнатылған. Жоғары ретті дифференциалдық теңдеулер үшін бөлінбеген көпнүктелі-интегралдық шарттары бар есептер әулеттерінің шешімдерін табу алгоритмдері құрылған және олардың бірмәнді шешілімділігі шарттары бастапқы берілімдер терминінде орнатылған.

Зерттеу нәтижелерінің теориялық маңызы бар және көптіректі арқалықтар теориясында, математикалық биология есептерінде қолданылуы мүмкін, сонымен бірге

жоғарғы ретті дифференциалдық теңдеулер үшін қолданбалы есептерді математикалық моделдеу кезінде пайдаланылуы мүмкін.

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**INTRODUCTION**

Report contains researches on the theory of problems with non-separated multipoint-integral conditions for high-order differential equations.

One of the important and actively developing areas of the qualitative theory of differential equations are boundary value problems which connect with numerous applications in physics, chemistry, biology and other applied problems.

Mathematical modeling of many processes of the theory of oscillations, the theory of impulse systems, and the theory of multi-support beams lead to problems with multipoint and integral conditions for differential equations of various orders.

In recent years, researchers have increased their attention to boundary value problems with multipoint and integral conditions for high-order differential equations. This is due to the specifics of the functioning of measuring instruments, with the use of which measurements are non-local in nature. Namely, the measurements are carried out not instantly, but over a certain period of time, and measurements at a single point actually characterize the state of the object as a whole in a certain area containing the measurement point. Studies of boundary value problems with multipoint and integral conditions began at the beginning of the 20th century (see [1–19] and the bibliography therein) and became more active with the advent of the work of many authors for both equations with ordinary and partial derivatives [4, 16 , 31-34]. The main apparatus for studying and solving problems with multipoint and integral conditions remains the method of Green functions, the construction of which is very difficult, due to the complexity of the object and insufficient knowledge of its properties. One of the ways to overcome these difficulties is to develop constructive research methods and solve problems with multipoint and integral conditions for systems of differential equations that do not use the fundamental matrix and Green's function. In this regard, to solve some classes of boundary value problems with multipoint and integral conditions, there were proposed methods that use numerical algorithms of the sweep method, shift conditions [1-2, 16, 35-38]. In most cases, boundary value problems with integral conditions based on the introduction of new variables and an increase in the number of differential equations were reduced to problems with multipoint conditions. This led to the fact that special methods for solving problems with non-separated multipoint and integral conditions for differential equations were hardly developed before. The overdetermined multipoint problems in which the boundary conditions in the intermediate points are "superfluous" turned out to be poorly studied. Such problems are directly related to spline theory, similar to the Vallee-Poussin problem to interpolation theory, and are also used in the theory of multi-support beams [9, 24-32, 37-38].

The current state of computing and information technologies requires the use of constructive methods for numerical and approximate solution of problems with non-separated multipoint-integral conditions for differential equations. Constructive methods allow, along with the establishment of solvability conditions, to give algorithms for finding approximate solutions that arbitrarily closely approximate the exact solution of the problem under study[21-23]. Therefore, it becomes necessary to create new and develop effective constructive methods for studying problems with non-separated multipoint-integral conditions for high-order differential equations.

The relevance of the Project is due, on the one hand, to the importance of the practical application of problems with non-separated multipoint-integral conditions for high-order differential equations in the study of various processes of the surrounding world, on the other hand, it is due to the need to develop constructive methods to construct solutions to problems with non-separated multipoint-integral conditions.

In the work proposes algorithms of the Dzhumabaev parametrization method for finding of solutions to the problem with non-separated multipoint-integral conditions for high-order differential equations. Conditions for the convergence of algorithms of the Dzhumabaev parametrization method for finding of solutions to the problem with non-separated multipoint- integral conditions for high-order differential equations are determined. Conditions for the unique solvability of the problem with non-separated multipoint-integral conditions for high-order differential equations are established in the terms of the initial data.

The coefficient conditions for the unique solvability of the problems under study for high-order differential equations established in the work, as well as the developed approximate methods, are a significant contribution to the theory of boundary-value problems for high-order differential equations.

The fundamental difference between ideas and the scientific significance of the results of work from existing analogues is the development of algorithms for the parameterization method for solving problems with non-separated multipoint-integral conditions for high-order differential equations; the establishment of coefficients solvability conditions for problems with non-separated multipoint-integral conditions for high-order differential equations.

The applied methodology for scientific research is assessed by high quality: the Dzhumabaev parameterization method and its modifications is used. The importance of the developed approach and the value of the results obtained are confirmed by the publications of the project leader and performers in high-ranking periodical journals with the impact factor of the near and far abroad.

The tasks set in the project for 2020-2021 have been completed. The strengths of the project include: the parameterization method developed by prof. D.S.Dzhumabaev and its modifications; the results were tested at scientific conferences; scientific results on the conducted research were accepted for publication in rating periodical journals with an impact factor.

The results obtained by the executors are new and make a significant contribution to the modern theory of boundary value problems for high-order differential equations.

The research carried out and the results correspond to the calendar work plan of scientific research works on the project № AP08955461: "Parameterization method to solve problems with non-separated multipoint-integral conditions for high-order differential equations" for 2020-2021 years of The specialized scientific field8.1: «Fundamental and applied research in mathematics and mechanics» of The priority area8: «Research in the field of natural sciences».

In the present report reflects research on the project "Parameterization method to solve problems with non-separated multipoint-integral conditions for high-order differential equations" for 2020-2021 years. The results obtained are a further development of the theory of boundary value problems for high-order differential equations.

For 2020, an interim report on scientific research work was prepared on the project “ Parameterization method to solve problems with non-separated multipoint-integral conditions for high-order differential equations ”, 2020 report inventory number: 0220РК01652.

**MAIN PART OF THE REPORT ON SRW**

Choice of research direction:

- Construction of algorithms for the parameterization method and establishment of conditions for the solvability of problems with non-separated multipoint-integral conditions for high-order differential equations;

- Construction of new general solutions of high-order differential equations, establishment of their properties and solvability conditions for problems with non-separated multipoint-integral conditions.

**1 Parameterization method to solve problem with non-separated multipoint-integral**

**conditions for high-order differential equations**

**1.1 Algorithms of parameterization method for finding of solution to problem with non-separated multipoint-integral conditions for high-order differential equations**

A problem with non-separated multipoint-integral conditions for high-order differential equations is investigated by the Dzhumabaev parametrization method. On its basis algorithms for finding a solution to a problem with non-separated multipoint-integral conditions for high-order differential equations are constructed.

The results of the section correspond to point 1.1 of the Calendar work plan for 2020.

On the interval  consider problem with non-separated multipoint-integral conditions for high-order differential equations

, (1.1)



, , (1.2)



where is unknown function, the functions , , and are continuous on , are constants, the functions are continuous on , , , , , are constants, .



Let be the space of continuous on functions with norm .



A solution to problem (1.1), (1.2) is called the function , having derivatives , , it satisfies to differential equation (1.1) for all and multipoint-integral conditions (1.2).



For investigating problem (1.1), (1.2) it is applied the parameterization method [20].

Let us give a scheme of the method without dividing the segment .

Introduce the following notations:

, , , …, ,



here are unknown parameters, .



In problem (1.1), (1.2) we make the replacement of functions:

, , ,…, ,

where  is a new unknown function.

Then problem (1.1), (1.2) is reduced to the following equivalent problem

, (1.3)

, , , …, , (1.4)

, (1.5)

.

So, we obtained boundary value problem with parameters for high-order differential equation (1.3)-(1.5).

A solution to problem (1.3)-(1.5) is a pair , where the function  has derivatives , , the parameter , satisfies to high-order differential equation with parameters (1.3) for all , initial conditions (1.4) and multipoint-integral conditions (1.5).

At fixed values of parameters , , problem (1.3), (1.4) is Cauchy problem for high-order differential equation. For determining unknown parameters , , we have the relations (1.5), consisting on  multipoint-integral conditions.

Problems (1.1), (1.2) and (1.3)-(1.5) are equivalent.

Consider Cauchy problem (1.3), (1.4). Let us take the function  as the required function and set

, . (1.6)

Taking into account initial conditions (1.4), we obtain

. (1.7)

Differentiating it times by , we find



,

. (1.8)

Substituting the obtained expressions into (1.3), for  we obtain Volterra integral equation of second kind

, , (1.9)

where the kernel  and the function  are determined by the following expressions

, , , (1.10)

, ,

(1.11)

respectively.

The function  is continuous on , depends on coefficients of equation (1.3) and the introduced unknown parameters , .

Substituting the corresponding values of the function  and its derivatives from expressions (1.7), (1.8) for , , , into conditions (1.5), we obtain a system of equations with unknown parameters :







,

. (1.12)

Grouping the coefficients corresponding to the same parameters , , in the left-hand sides of the algebraic equations (1.12), we compose the -matrix  and write the system of algebraic equations (1.12) in the vector form



, (1.13)

where , , , , .

Assume that matrix  is invertible. Then vector defines from the system of equations (1.13) in the following form:



.

The existence of the inverse to the matrix  ensures the compatibility of conditions (1.4) and (1.5). This allows us to consider problem (1.3), (1.4), (1.5) as a problem with initial and non-separated multipoint-integral conditions for a high-order ordinary differential equation with additional parameters.

If the parameters , , are known, from the Volterra integral equation of second kind (1.9) we find the function  for all . Then, substituting the parameters , , and the found function  into representations (1.7) and (1.8), we define the function  and its derivatives for all . Further composing sums

, , ,…, ,

we find a solution to original problem (1.1), (1.2).

If the function  is known, from the system of algebraic equations (1.13) we find the parameters , . Then, substituting the function  and the found parameters ,  in representations (1.7) and (1.8), we define the function and its derivatives for all . Then, composing sums

, , ,…, , we find a solution to original problem (1.1), (1.2).

Since both functions ,  and parameters , , are unknown, an iterative process is used. Successive approximations of the pairs  and ,  are determined from the following algorithm.

Step 0. 1) Assuming  in the right-hand side of (1.13) and solving system of algebraic equations, we find an initial approximation . 2) Solving integral equation (1.9) for , , we find  for all . 3) Substituting instead of  and , , in integral representations (1.7) and (1.8), founded  and , , respectively, we define initial approximation  and its derivatives , , for all .

And so on.

Step. 1) Assuming  in the right-hand side of (1.13) and solving system of algebraic equations, we find . 2) Solving integral equation (1.9) for , , we find  for all . 3) Substituting instead of  and ,  in integral representations (1.7) and (1.8), founded  and , , respectively, we define function  and its derivatives , , for all . 

As you can see from the algorithm, it includes three stages: Stage 1. Solve system of algebraic equations (1.13) with respect to parameters , ; Stage 2. Solve Volterra integral equation of second kind (1.9) with respect to function  for all ; Stage 3. Through integral representations (1.7) and (1.8) determine function  and its derivatives , , for all .

The condition for the realizability of the algorithm is the invertibility of the matrix . Now it is important to clarify the conditions for the convergence of the proposed algorithm, which ensure uniform convergence for all  sequences of pairs  (and functions ) as  to a pair  is the solution of problem (1.3) - (1.5) (and  is the solution of the integral equation (1.9) for , ).

**1.2 Conditions of the convergence of algorithms for finding of solutions to problem with non-separated multipoint-integral conditions for high-order differential equations**

The results of the section correspond to point 1.2 of the Calendar work plan for 2020.

Let , , , .

The following statement gives conditions for the convergence of the algorithm proposed in Section 1.1, which simultaneously guarantee the existence of a unique solution to problem (1.3) - (1.5).

Theorem 1.1. Let the matrix  be invertible and the following inequalities are true:

1. , where  is positive constant;
2. 

.

Then successive approximations ,  and , defining by algorithm, are uniformly converge to ,  and , respectively, for all , and the pair  is a unique solution to problem (1.3)-(1.5).

The proof is carried out according to the scheme of the proposed algorithm.

From the equivalence of problems (1.3)-(1.5) and (1.1), (1.2) it follows that the function , , be a solution to problem (1.1), (1.2).

We have

Theorem 1.2. Let the matrix  be invertible and the inequalities 1), 2) of Theorem 1.1 are fullfilled.

Then function , defining as sum of functions  for all  is a unique solution to problem (1.1), (1.2).

The proofs of the theorems are carried out on the basis of the above algorithm and are presented in the work from «Kazakh Mathematical Journal» published in 2020.

**1.3 Conditions for the unique solvability to problem with non-separated multipoint-integral conditions for high-order differential equations**

The results of the section correspond to point 1.3 of the Calendar work plan for 2020.

Consider problem (1.3)-(1.5).

Set , , ,.

The function  is called -th iteration of kernel .

If we denote greatest value  on  by , then the following estimate is true . Then the series  is uniformly converges on  and integral equation (1.9) has a unique solution



 . (1.14)

Here  is a resolvent for integral equation (1.9).

Substituting the expression for  from (1.14) into relation (1.7), we obtain the representation of the desired solution  in the following form







. (1.15)

Similarly, we define its derivatives with respect to :









, . (1.16)

So, the desired solution  and its derivatives are explicitly expressed through the introduced unknown parameters , . Substituting the corresponding values of the function and its derivatives from expressions (3.2), (3.3) for , ,  in conditions (1.5), we obtain a system of equations with unknowns:



,  . (1.17)

If the matrix  is invertible, then vector is determined from the system of equations (1.17) in the following form: .



The existence of the inverse to the matrix  ensures the compatibility of conditions (1.4) and (1.5). This allows us to consider problem (1.3), (1.4), (1.5) as a problem with non-separated multipoint-integral conditions for a high-order ordinary differential equation with additional parameters. Substituting the components , , of the found vector into expression (1.15), we obtain a representation of the desired solution to problem (1.1), (1.2).



The above statement implies a statement establishing sufficient conditions for the existence of a unique solution to the problem with non-separated multipoint-integral conditions (1.1), (1.2).

Theorem 1.3. Let the matrix  on dimension be invertible.



Then problem with non-separated multipoint-integral conditions for high-order ordinary differential equation (1.1), (1.2) has a unique solution.

Thus, the conditions for the unique solvability of problem (1.1), (1.2) are given in terms of the matrix  composed using the resolvent of the Volterra integral equation of second kind (1.9). The kernel and the right-hand side of the integral equation (1.9) are determined in terms of the coefficients of the differential equation (1.1) and the multipoint-integral condition (1.2). This allows us to assert that the conditions for the solvability of problem (1.1), (1.2) are established in terms of the initial data. The solution of the integral equation (1.9) is an important issue, and the construction of its approximate solutions is based on the calculation of iterated kernels .

**2 New general solution of high-order differential equations and its application to solve problem with non-separated multipoint-integral conditions**

**2.1 New general solution of high-order differential equations and its properties**

On the basis of the parametrization method, the definition of a new general solution of high-order differential equations is introduced and its properties are investigated.

The results of the section correspond to point 2.1 of the Calendar work plan for 2021.

On interval  consider the problem with non-separated multipoint-integral conditions for high-order differential equations

, (2.1)

, , (2.2)

where  is unknown function, the functions , , and  are continuous on ,  are constants,  are continuous on  functions, , , , ,  are constants, .

Let  be the space of continuous on  functions  with norm .

A solution to problem (2.1), (2.2) is called a function , having derivatives , , satisfies to differential equation (2.1) for all  and multipoint-integral conditions (2.2).

By introducing a new functions

, , , … , ,

the problem (2.1), (2.2) is reduced to problem for system of differential equations

, , , (2.3)

, , (2.4)

where  is unknown vector function, the -matrices ,  and  vector function  have the form

, 



and are continuous on , the -matrices  and  vector  have the form

, ; .

For solve problem is applied Dzhumabaev parametrization method [20] with a partition on interval .

Using points  , , we make a partition : .

Let  be the space of system functions , where the functions  are continuous and have finite left-hand limits  for all  with norm .

Let the vector function  be a solution to system (2.3) and  be its restriction on sub-interval , i.e. , , . Then the system functions  belongs to , and its elements , , satisfy to system of differential equations

, , . (2.5)

Further, we introduce parameters , .

Making a replacement  on each th interval , we obtain a system of differential equations with parameters

, , , (2.6)

and initial conditions

, , (2.7)

where  is -dimensional null vector.

For fixed parameter  and  Cauchy problem (2.6), (2.7) has a unique solution and system functions  belongs to . System functions  is called a solution of Cauchy problem (2.6), (2.7).

We introduce a new definition of general solution to system of differential equations (2.3) and original system (2.1).

Definition 2.1. Let  be the solution of Cauchy problem

(2.6), (2.7) for parameter . Then function , determining by equalities  for , , and , is called the general solution to system (2.3).

Definition 2.2. Let  be the solution of Cauchy problem

(2.6), (2.7) for parameter  with elements  and , respectively, for . Then the function  and its derivatives , , determining by equalities

, , , ,

, , , is called the general solution to high-order differential equation (2.1).

Let  be a fundamental matrix of system of differential equations

,  , , ,

where  is -dimensional unit square matrix.

The solution of Cauchy problems with parameters (2.6), (2.7) write in the next form:

 , . Consider the Cauchy problems on the sub-intervals

,  , , , (2.8)

where  is square matrix or vector on dimension  and continuous on .

Denote by  a unique solution of Cauchy problem (2.8) on each r-th interval. From uniqueness of solution to Cauchy problem for linear system of differential equations it follows that

 , .

So, we can represent  general solution to system (2.3) in the next form:

, , , (2.9)

, . (2.10)

The following statements show the validity of functions  and  as a general solution.

Theorem 2.1. Let be given piecewise-continuous on  function  with possible discontinuities at the points , , and  is  general solution to system (2.3). Assume that a function  has continuous derivative and satisfies to system (2.3) for all . Then exists a unique  such that the equality  holds for all .

Theorem 2.2. Let be given piecewise-continuous on  function  with possible discontinuities at the points , , and  is  general solution to high-order differential equation (2.1). Assume that a function  has continuous derivatives up to th order and satisfies to high-order differential equation (2.1) for all . Then exists a unique  with elements , , such that the equality  holds for all .

Lemma 2.1. Let  be a solution to system (2.3) and  is  general solution to system (2.3). Then exists a unique  such that the equality  for all .

Lemma 2.2. Let  be a solution to high-order differential equation (2.1) and  is  general solution to high-order differential equation (2.1). Then exists a unique  with elements , , such that the equality  holds for all .

If the  is the solution to system (2.3) and  is system functions composed by its restrictions on sub –intervals , , then the following equations

,  (2.11)

are valid. These equations are the continuity conditions of solution to system (2.3) at the interior points of partition .

Theorem 2.3. Let the system functions  be belong to . Assume that functions , , satisfy to system (2.5) and the continuity conditions (2.11).

Then the function  given by equalities , , , and , is continuous on  and continuously differentiable on , and satisfies to system (2.3).

From Theorem 2.3 it follows that function  given by equalities

, , , и ,

has derivatives up to -th order on , defining by equalities

, , , and

, ,

has continuous derivative of -th order on , and satisfies to high-order differential equation (2.1).

**2.2 Algorithms for finding of solution to problem with non-separated multipoint-integral conditions for high-order differential equations**

Using new general solution the algorithms for finding of solution to problem with non-separated multipoint-integral conditions for high-order differential equations are constructed.

The results this section correspond to point 2.2 Calendar work plan on 2021.

The  general solution allow us the solvability to problem with non-separated multipoint-integral conditions to reduce the solvability to system of algebraic equations with respect to arbitrary vectors , . Substituting the corresponding expressions of the general solution to (2.9), (2.10), into the non-separated multipoint-integral conditions (2.4) and the continuity conditions (2.11), we obtain the system of algebraic equations



, (2.12)

, . (2.13)

Denote by  the  - matrix, corresponding to left-hand side of system (2.12), (2.13) and rewrite system in the next form

, , (2.14)

where .

For any partition  Theorems 2.1 and 2.3 ensure the validity of the following statement.

Lemma 2.3. If the function  is a solution to the problem with non-separated multipoint-integral conditions (2.3), (2.4) and , , then the vector  is a solution to system (2.14). Conversely, if the vector  is a solution to system (2.14) and  is a solution of Cauchy problem (2.6), (2.7) for parameter , then the function  given by equalities

 for , , and , is a solution to the problem with non-separated multipoint-integral conditions (2.3), (2.4).

We propose the following algorithm for finding of solutions to the problem with non-separated multipoint-integral conditions (2.3), (2.4):

1. Using points  , , we make the partition .

2. Using a restrictions  of vector function  on sub-intervals , we introduce a parameters , . Making a replacement  on each interval , we obtain an equivalent problem with parameters, , , (2.15)

, , (2.16)

, (2.17)

, . (2.18)

3. Construct a fundamental matrix  of system of differential equations

,  , , .

4. A solution of Cauchy problems with parameters (2.15), (2.16) is presented by the fundamental matrix :

 , . (2.19)

5. Introduce the  general solution to system (2.3).

6. On the basis representation of the  general solution it is composed system of algebraic equations (2.14).

7. Assuming the invertibility of the  - matrix  we determine a unique solution to system (2.14): .

8. Using components of founded parameter we define a unique solution of Cauchy problems (2.15), (2.16) are functions , , .

9. By equalities  for , , and , we determine a function  is a unique solution to problem (2.3), (2.4).

10. First component of the vector  is the function , be a solution to problem (2.1), (2.2): , and the remaining components are a derivatives of the function  up to -th order on .

**2.3 Criteria of the unique unique solvability to problem with non-separated multipoint-integral conditions for high-order differential equations**

Conditions for the unique solvability of the problem with non-separated multipoint-integral conditions for high-order differential equations are established in the initial data.

The results this section correspond to point 2.3 Calendar work plan on 2021.

Definition 2.3. Problem with non-separated multipoint-integral conditions for a system of differential equations (2.3), (2.4) is called a unique solvable, if for any pair , where  and , it has a unique solution.

Definition 2.4. Problem with non-separated multipoint-integral conditions for a high-order differential equations (2.1), (2.2) is called a unique solvable, if for any , , , it has a unique solution.

Lemma 2.3 and well-known theorems of linear algebra establich the validity of the following statements.

Theorem 2.4. Problem with non-separated multipoint-integral conditions for the system of differential equations (2.3), (2.4) is solvable if only if, then the vector  orthogonals to the kernel of the transposed matrix , i.e. the equality  holds for all , where  is scalar product in .

Theorem 2.5. Problem with non-separated multipoint-integral conditions for the system of differential equations (2.3), (2.4) is uniquely solvable if only if, then the  - matrix  is invertible.

From the equivalence of problems (2.1), (2.2) and (2.3), (2.4) it follows

Theorem 2.6. Problem with non-separated multipoint-integral conditions for the high-order differential equations (2.1), (2.2) is solvable if only if, then the vector  orthogonals to the kernel of the transposed matrix , i.e. the equality  holds for all , where  is scalar product in .

Theorem 2.7. Problem with non-separated multipoint-integral conditions for the high-order differential equations (2.1), (2.2) ) is uniquely solvable if only if, then the  - matrix  is invertible.

**2.4 Method of solving to problem with non-separated multipoint-integral conditions for high-order differential equations**

An effective method for solving to problem with non-separated multipoint-integral conditions for high-order differential equations is developed.

The results this section correspond to point 2.4 Calendar work plan on 2021.

Based on the results of Sections 2.1-2.3, we propose an effective method for finding a solution to problem (2.1), (2.2).

Stage 1. Solve Cauchy problems on sub-intervals

,  , ,

,  , ,

and find  and , .

Stage 2. Using founded matrices and vectors we compose a system of linear algebraic equations (2.14).

Stage 3. Solve the constructed system and find . Elements of  are values of solution to problem (2.3), (2.4) at the left-hand points of sub-intervals: , .

Stage 4. Solve Cauchy problems

,  , ,

and determine values of solution  at the remaining points of sub-intervals.

Stage 5. Using founded the values of solution , we determine values of solution  to problem with non-separated multipoint-integral conditions for high-order differential equations (2.1), (2.2).

As follows that from Lemma 2.3, any solution to system (2.14) is determined the values of solution to problem (2.3), (2.4) at the initial points of sub-intervals.

The accuracy of the proposed algorithm of the method depends on the accuracy of the calculated coefficients and the right-hand sides of system (2.14). Cauchy problem for the system of differential equations is the main problem of the proposed algorithm of the method. Choosing an approximate method for solving this problem, we give an approximate method for solving problem (2.3), (2.4).

Solving Cauchy problems by numerical methods, we propose the numerical methods for solving

the problem with non-separated multipoint-integral conditions (2.3), (2.4). These methods allow us to find numerical solutions of problem with non-separated multipoint-integral conditions for high-order differential equations (2.1), (2.2).

**2.5 New general solution of a family high-order differential equations and its properties**

A new general solution of families of high-order differential equations is constructed and its properties are established.

The results of the section correspond to point 2.5 of the Calendar work plan for 2021.

On the domain  consider a problem with non-separated multipoint-integral conditions for the familyhigh-order differential equations

, (2.20)

, , , (2.21)

where  is unknown function, the functions , , and  are continuous on , the functions ,  and  are continuous on  and , respectively, , , , .

Let  be the space of continuous on  functions  with norm

.

A solution to problem (2.20), (2.21) is called the function , having derivatives , , satisfies to the family of differential equations (2.1) for all  and multipoint-integral conditions (2.21).

Problem (2.20), (2.21) by introducing new functions

, , , ..., , reduce to a family of problems for a system of differential equations

, , , (2.22)

, , , (2.23)

where  is unknown vector function, the  -matrices ,  and the  vector  have the next form

, 

the  -matrices  and the  vector  have the form

, ;

, ,

and are continuous on , .

Using the lines  , , we make a partition  of domain : , , , .

Let  be the space of systems functions , where the functions  are continuous and have a finite left-hand side limits  uniformly with respect to  for all  with norm .

Let vector function  be the solution to family systems (2.22) and  is its restriction to sub-domain , . Then the system functions  belongs to , and its elements , , satisfy to family systems of differential equations

, , . (2.24)

Further, introduce a functional parameters , , . Making a replacement  on each sub-domain , we obtain the family systems of differential equations with parameters

, , , (2.25)

and initial conditions

, , . (2.26)

For fixed parameter  and  the family Cauchy problems (2.25), (2.26) has a unique solution , and the system functions  belongs to . System functions  is called a solution to the family Cauchy problems (2.25), (2.26).

A new definition of general solutions to family systems of differential equations (2.22) and original system (2.20) are introduced.

Definition 2.4. Let  be a solution to families Cauchy problems (2.25), (2.26) for functional parameter  . Then a function , defining by equalities  for , , and , is called a general solution to family systems (2.22).

Definition 2.5. Let  be a solution to families Cauchy problems (2.25), (2.26) for parameter  with elements  and  , respectively, for . Then the function  and its derivatives , , determining by equalities

, ,

, , , , , ,

is called a general solution to family of high-order differential equations (2.20).

Let  be a fundamental matrix of family systems of differential equations ,  , , .

Solution to family Cauchy problems with parameters (2.25), (2.26) rewrite in the next form:

 , .

Consider the families Cauchy problems on the sub-domains

,  , , , (2.27)

where  is square matrix or vector on dimension  and continuous on .

Denote by  a unique solution to families Cauchy problems (2.27) on each sub-domain. From uniqueness of solution to families Cauchy problems for linear system of differential equations it follows

 , .

So, we can represent the  general solution to system (2.3) in the following form:

, , , (2.28)

, . (2.29)

Theorem 2.8. Let be given piecewise-continuous on  function  with possible discontinuities at the lines , , and  is the  general solution to system (2.22). Assume that the function  has continuous partial derivative by  and satisfies to family systems (2.22) for all . Then exists a unique  such that the equality  holds for all .

Theorem 2.9. Let be given piecewise-continuous on  function  with possible discontinuities at the lines , , and  is the  general solution to family of high-order differential equations (2.20). Assume that the function  has continuous partial derivatives up to -th order by  and satisfies to family of high-order differential equations (2.20) for all . Then exists a unique  with elements , , such that the equality  holds for all .

Lemma 2.4. Let  be solution to family systems (2.22) and  is the  general solution to system (2.22). Then exists a unique  such that the equality  holds for all .

Lemma 2.5. Let  be solution to the family high-order differential equations (2.20) and  is the  general solution to the family high-order differential equations (2.20). Then exists a unique  with elements , , such that the equality  holds for all .

If the  is solution to family systems (2.22) and  is system functions composed by its restrictions on the sub=domains , , then the following equations

, ,  (2.30)

are valid. These equations are the continuity conditions of solution to family systems (2.22) at the interior lines of the partition .

Theorem 2.10. Let the system functions  be belongs to . Assume that the functions , , satisfy to family systems (2.24) and the continuity conditions (2.30).

Then the function , given by equalities

, , , and , is continuous on , has a continuous partial derivative in  on , and satisfies to family systems (2.22).

From Theorem 2.10 it follows that function , given by equalities

, , , and , ,

has a partial derivatives up to -th order in  on , defining by equalities

, , , and , , continuous partial derivative of -th order in  on , and satisfies to family high-order differential equations (2.20).

**2.6 Algorithms for finding of solution to families of problems with non-separated**

**multipoint-integral conditions for high-order differential equations and conditions for their unique solvability**

Algorithms for finding solutions to families of problems with non-separated multipoint-integral conditions for high-order differential equations are constructed and conditions for unique solvability are established in the terms of initial data.

The results of the section correspond to point 2.6 of the Calendar work plan for 2021.

The  general solution allow us the solvability of family problems with non-separated multipoint-integral conditions to reduce a solvability of system linear functional equations with respect to arbitrary vectors , . Substituting the corresponding expressions of the  general solution (2.28), (2.29), into the non-separated multipoint-integral conditions (2.23) and the continuity conditions (2.30), we obtain the system of functional equations



, (2.31)

, , . (2.32)

Denote by  the  - matrix, corresponding to left-hand part of system (2.31), (2.32) and rewrite system in the next form

, , , (2.33)

where  .

For any partition  Theorems 2.8 and 2.10 ensure the validity following statement.

Lemma 2.6. If  is solution to family problems with non-separated multipoint-integral conditions (2.22), (2.23) and , , , then the vector  is a solution to family systems (2.33). Conversely, if  is the solution to family systems (2.33) and  is the solution family Cauchy problems (2.25), (2.26) for parameter , then the function  given by equalities  for , , and , , is a solution to families problems with non-separated multipoint-integral conditions (2.22), (2.23).

An algorithm for finding solutions to problem with non-separated multipoint-integral conditions (2.22), (2.23) is proposed:

1. Using the lines  , , we make the partition .

2. Using the restrictions  of vector function  on sub-domain , we introduce parameters , , . Making a replacement  on each sub-domain , we obtain an equivalent family problem with functional parameters.

3. Construct the fundamental matrix  of family systems of differential equations.

4. Solution to family Cauchy problems with parameters (2.25), (2.26) represents through fundamental matrix :



, .

5. Introduce the  general solution to family systems (2.22).

6. On the basis the expression of the  general solution is composed system of functional equations (2.33).

7. Assuming the invertibility of the  - matrix  for all , we define a unique solution to system (2.33): .

8. Using components of founded parameter , we determine a unique solution to family Cauchy problems (2.25), (2.26) - are functions , , .

9. By equalities  for , , and , it is defined the function  - is the unique solution to family problems (2.22), (2.23).

10. First component of vector  is the function , be a solution to family problems (2.20), (2.21): , and the remaining components are partial derivatives of  up to -th order in  on .

Definition 2.6. A family of problems with non-separated multipoint-integral conditions for the system of differential equations (2.22), (2.23) is called uniquely solvable if for any pair , where  , , it has a unique solution.

Definition 2.7. A family of problems with non-separated multipoint-integral conditions for the high-order differential equation (2.20), (2.21) is called uniquely solvable if for any , , , it has a unique solution.

Theorem 2.11. The family of problems with non-separated multipoint-integral conditions for the system of differential equations (2.22), (2.23) is solvable if only if, then the vector  orthogonals to the kernel of the transposed matrix , i.e. the equality  holds for all , , where  is scalar product in .

Theorem 2.12. The family of problems with non-separated multipoint-integral conditions for the system of differential equations (2.22), (2.23) is unique solvable if only if, then the  - matrix  is invertible for all .

From the equivalence of problems (2.20), (2.21) and (2.22), (2.23) it follows

Theorem 2.13. The family of problems with non-separated multipoint-integral conditions for the high-order differential equation (2.20), (2.21) is solvable if only if, then the vector  orthogonals to the kernel of the transposed matrix , i.e. the equality  holds for all , , where  is scalar product in .

Theorem 2.14. The family of problems with non-separated multipoint-integral conditions for the high-order differential equation (2.20), (2.21) is unique solvable if only if, then the  - matrix  is invertible for all .

**CONCLUSION**

This report contains research on the topic " Parameterization method to solve problems with non-separated multipoint-integral conditions for high-order differential equations" performed in 2020-2021 in the field of problems with non-separated multipoint-integral conditions for high-order differential equations and mathematical modeling. The results obtained are aimed at further development of the theory of boundary value problems of high-order differential equations.

Algorithms of the method of parametrization of finding a solution to a problem with non-separated multipoint-integral conditions for high-order differential equations are constructed. Conditions for the convergence of algorithms for finding solutions to a problem with non-separated multipoint-integral conditions for high-order differential equations are determined. Conditions for the unique solvability of the problem with non-separated multipoint-integral conditions for high-order differential equations are established.

A new general solution of high-order differential equations is constructed and its properties are established. Algorithms for finding solutions to a problem with non-separated multipoint-integral conditions for high-order differential equations are proposed using a new general solution. Conditions for the unique solvability of the problem with non-separated multipoint-integral conditions for high-order differential equations are established in terms of the initial data. An effective method for solving a problem with non-separated multipoint-integral conditions for high-order differential equations is developed. The results are extended to families of problems with non-separated multipoint-integral conditions for high-order differential equations.

The research work done during the reporting period on the topic concerns topical problems of the modern theory of boundary value problems for differential equations and its applications. The results obtained by the performers of the theme are new and sufficiently fully reflect the content of the tasks.

The high level of performed research works is characterized by publications, which is reflected in Appendix A - in the list of published works of this report. The calendar work plan on the project for 2020-2021 is given in Appendix B.

**LIST OF USED SOURCES**

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**APPENDIX А**

**List of publications**

2020 year

In domestic publications

1Imanchiyev A.E., Ermek A.A. Parameterization method for solving problem with non-separated multipoint-integral conditions for the differential equations high order // Kazakh Mathematical Journal. – 2020. - Том 20. No 4. – C. 74-86. (КазБЦ, КОКСОН МОН РК)

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2021 year

In foreign publications

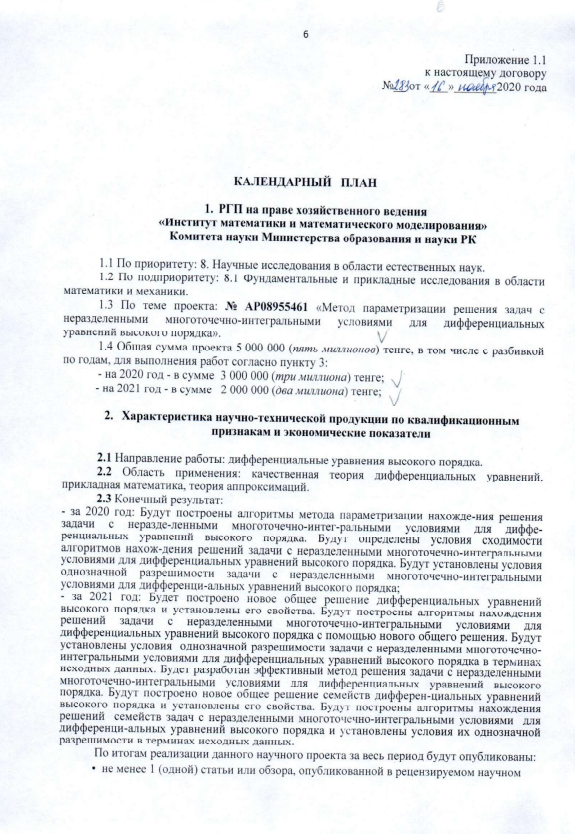
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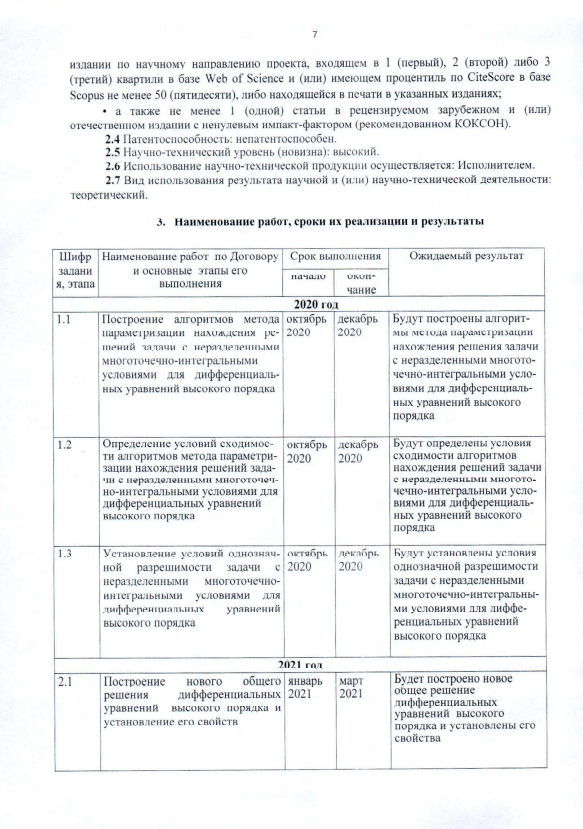
DOI: 10.1134/S0012266121010092.

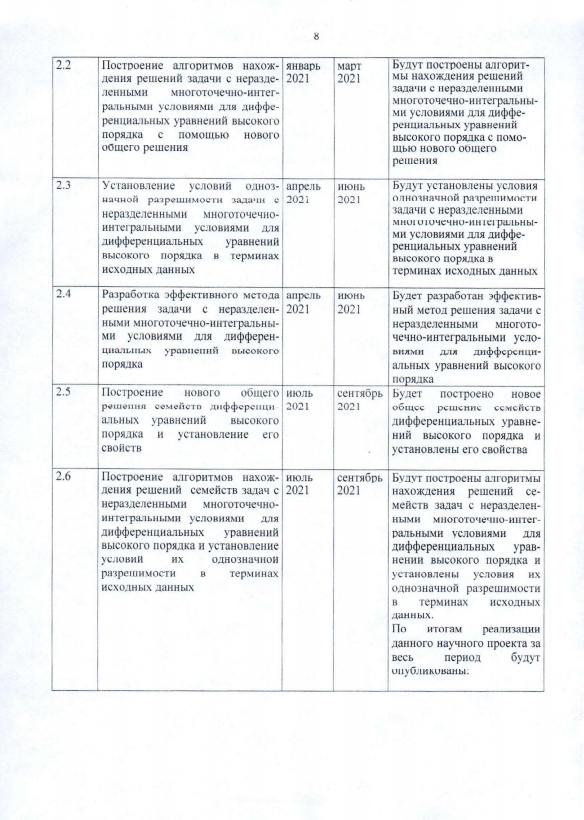
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**APPENDIX B**

**Calendar work plan**









**Translation into English of the calendar work plan of the grant project**

**"Parameterization method to solve problems with non-separated multipoint-integral conditions for high-order differential equations"**

Appendix 1.1

to Agreement № from 2020

for grant funding

**TECHNICAL SPECIFICATION AND WORK SCHEDULE**

Under agreement № 283 from November 16, 2020

1. **RSE on the right of economic management “Institute of mathematics and mathematical modeling” of the Committee of science of the Ministry of education and science of the Republic of Kazakhstan**
   1. By priority: 8 Scientific research in the field of natural sciences
   2. By sub-priority: 8.1. Fundamental and applied research in the field of mathematics and mechanics.
   3. On the theme of the project: No. AP08955461 “Parameterization method to solve problems with non-separated multipoint-integral conditions for high-order differential equations”
   4. The total amount of the project is 5 000 000 (five million) tenge, including with a breakdown by years, for implementing the works according to item 2:

* for 2020 – 3 000 000 (three million) tenge;
* for 2021 - 2 000 000 (two million) tenge.

1. **Characteristics of scientific and technical products by qualification features and economic indicators**
   1. Direction of the work: high-order differential equations.
   2. Scope: the qualitative theory of differential equations, applied mathematics, the theory approximations.
   3. Final result:

* for 2020: Algorithms of the parametrization method will be constructed for finding a solution to the problem with non-separated multipoint-integral conditions for high-order differential equations. The conditions for the convergence of algorithms for finding solutions to the problem with non-separated multipoint-integral conditions for high-order differential equations will be determined. The conditions for the unique solvability of the problem with non-separated multipoint-integral conditions for high-order differential equations will be established;
* for 2021: A new general solution of high-order differential equations will be constructed and its properties established. Using a new general solution, algorithms for finding solutions to the problem with non-separated multipoint-integral conditions for high-order differential equations will be constructed. The conditions for the unique solvability of the problem with non-separated multipoint-integral conditions for high-order differential equations will be established in terms of initial data. An effective method will be developed for solving a problem with non-separated multipoint-integral conditions for high-order differential equations. A new general solution of families of high-order differential equations will be constructed and its properties established. Algorithms for finding solutions to families of problems with non-separated multipoint-integral conditions for high-order differential equations will be constructed, and the conditions for their unique solvability in terms of the initial data will be established.

According to the results of the implementation of this project for the entire period:

- at least 1 (one) article or review published in a peer-reviewed scientific publication in the scientific direction of the Project, included in 1 (first), 2 (second) or 3 (third) quartiles in the Web of Science database and (or) having a CiteScore percentile in the Scopus database of at least 50 (fifty), or in print in the indicated editions;

* as well as at least 1 (one) article in peer-reviewed foreign and (or) domestic non-zero impact factor edition (recommended by CCSSE).
  1. Patentability: not patentable.
  2. Scientific and technical level (novelty): a high level.
  3. The use of scientific and technical products is carried out: by the Executor.

2.7 Type of use of the result of scientific and (or) scientific and technical activities: theoretical.

1. **Name of work, terms of their implementation and results**

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| Stage code | Name of works under the Contract and the main stages of its implementation | | Deadlines | | Expected results |
| Start | Ending |
| **2020** | | | | | |
| 1.1 | Construction of algorithms of the method of parameterization for finding solutions to problems with non-separated multipoint-integral conditions for high-order differential equations | | October 2020 | December 2020 | Algorithms of the parametrization method will be constructed for finding a solution to the problem with non-separated multipoint-integral conditions for high-order differential equations |
| 1.2 | Determination of conditions for convergence of algorithms of the parameterization method for finding solutions to problems with non-separated multipoint-integral conditions for high-order differential equations | | October 2020 | December 2020 | The conditions for the convergence of algorithms for finding solutions to the problem with non-separated multipoint-integral conditions for high-order differential equations will be determined |
| 1.3 | Establishment of conditions for the unique solvability of a problem with non-separated multipoint-integral conditions for high-order differential equations | | October 2020 | December 2020 | The conditions for the unique solvability of the problem with non-separated multipoint-integral conditions for high-order differential equations will be established |
| **2021** | | | | | |
| 2.1 | | Construction of a new general solution of high-order differential equations and the establishment of its properties | January 2021 | March 2021 | A new general solution of high-order differential equations will be constructed and its properties established |
| 2.2 | | Construction of algorithms for finding solutions to a problem with non-separated multipoint-integral conditions for high-order differential equations using a new general solution | January 2021 | March 2021 | Using a new general solution, algorithms for finding solutions to the problem with non-separated multipoint-integral conditions for high-order differential equations will be constructed |
| 2.3 | | Establishment of conditions for the unique solvability of a problem with non-separated multipoint-integral conditions for high-order differential equations in terms of initial data | April 2021 | June 2021 | The conditions for the unique solvability of the problem with non-separated multipoint-integral conditions for high-order differential equations will be established in terms of initial data |
| 2.4 | | Development of an effective method for solving a problem with non-separated multipoint-integral conditions for high-order differential equations | April 2021 | June 2021 | An effective method will be developed for solving a problem with non-separated multipoint-integral conditions for high-order differential equations |
| 2.5 | | Construction of a new general solution of families of high-order differential equations and the establishment of its properties | July 2021 | September 2021 | A new general solution of families of high-order differential equations will be constructed and its properties established |
| 2.6 | | Construction of algorithms for finding solutions to families of problems with non-separated multipoint-integral conditions for high-order differential equations and establishment of conditions for their unique solvability in terms of initial data | July 2021 | September 2021 | Algorithms for finding solutions to families of problems with non-separated multipoint-integral conditions for high-order differential equations will be constructed, and the conditions for their unique solvability in terms of the initial data will be established |

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| From customer:  Chairman of the State Institution "Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan"  \_\_\_\_\_\_\_\_\_\_\_\_\_\_ Kurmangalieva Zh.D.  м.п. | From the Contractor:  General Director of the RSE on the RK "Institute  of Mathematics and Mathematical Modeling" of the  Science Committee of the Ministry of Education  and Science of the Republic of Kazakhstan  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Sadybekov M.A.  м.п.  Familiarized with:  Scientific supervisor of the project  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Imanchiyev A.E. |