

**ABSTRACT**

Report 39 pp., 1 book, 19 sources, 2 annexes

INTEGRAL OPERATORS, THE KERNEL OF THE INTEGRAL OPERATOR, THE MARZINKEVICH-CALDERON THEOREM, LEBESGUE SPACES, LORENTZ SPACES, FOURIER TRANSFORM

The object of research is integral operators in Lebesgue spaces.

The aim of this work is to obtain for integral operators the strengthening of the Marcinkiewicz-Calderon interpolation theorem. Using interpolation methods, obtain inequalities of Hardy-Littlewood type and Nursultanov inequalities in weighted Lorentz spaces.

To achieve this goal, the following tasks are supposed to be solved:

1. In terms of the kernel of the integral operator, obtain the necessary conditions for the operator to be bounded in the Lebesgue space Lp for all ;

2. In terms of the kernel of the integral operator, obtain sufficient conditions for the operator to be bounded in the Lebesgue space Lp for all ;

3. Obtaining new inequalities of Hardy-Littlewood type for two-dimensional generalized Lorentz spaces.

4. Obtaining new inequalities of the Nursultanov type for two-dimensional generalized Lorentz spaces.

The research methods are based on the developments of the theory of interpolation, the theory of function spaces, the theory of integral operators.

The results obtained:

1. In terms of the kernel of the integral operator, necessary conditions are obtained for the operator to be bounded in the Lebesgue space Lp for all .

2. In terms of the kernel of the integral operator, sufficient conditions are obtained for the operator to be bounded in the Lebesgue space Lp for all ;

3. New inequalities of Hardy-Littlewood type are obtained for two-dimensional generalized Lorentz spaces.

4. New inequalities of the Nursultanov type are obtained for two-dimensional generalized Lorentz spaces.

3 papers were prepared: 2 papers were accepted for publication (Eurasian Mathematical Journal and Mathematical Notes), 1 paper was submitted to the editor (Eurasian Mathematical Journal).

**РЕФЕРАТ**

Отчет 39 с., 1 кн., 19 источн., 2 прил.

ИНТЕГРАЛЬНЫЕ ОПЕРАТОРЫ, ЯДРО ИНТЕГРАЛЬНОГО ОПЕРАТОРА, ТЕОРЕМА МАРЦИНКЕВИЧА-КАЛЬДЕРОНА, ПРОСТРАНСТВА ЛЕБЕГА, ПРОСТРАНСТВА ЛОРЕНЦА, ПРЕОБРАЗОВАНИЕ ФУРЬЕ

Обьектом исследования является интегральные оператор в пространствах Лебега.

Цель работы – для интегральных операторов получить усиления интерполяционной теоремы Марцинкевича-Кальдерона. Используя интерполяционные методы получить неравенства типа Харди-Литтлвуда и неравенства Нурсултанова весовых пространствах Лоренца.

Для достижения поставленной цели предполагается решение следующих задач:

1. В терминах ядра интегрального оператора получить необходимые условия для того, чтобы оператор был ограничен в пространстве Лебега Lp для всех ;
2. В терминах ядра интегрального оператора получить достаточные условия для того чтобы оператор был ограничен в пространстве Лебега Lp для всех ;
3. Получение новых неравенств типа Харди-Литтлвуда для двумерных обобщенных пространств Лоренца.
4. Получение новых неравенств типа Нурсултанова для двумерных обобщенных пространств Лоренца.

Методы исследования базируются на разработках теории интерполяции, теории функциональных пространств, теории интегральных операторов.

Полученные результаты:

В терминах ядра интегрального оператора получены необходимые условия для того, чтобы оператор был ограничен в пространстве Лебега Lp для всех .

В терминах ядра интегрального оператора получены достаточные условия для того чтобы оператор был ограничен в пространстве Лебега Lp для всех ;

Получены новые неравенства типа Харди-Литтлвуда для двумерных обобщенных пространств Лоренца.

Получены новые неравенства типа Нурсултанова для двумерных обобщенных пространств Лоренца.

Подготовлены 3 статьи: 2 статьи приняты в печать (Eurasian Mathematical Journal и Математические заметки), 1 статья сдана в редакцию (журнал Eurasian Mathematical Journal).

**CONTENT**

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**INTRODUCTION**

The real interpolation method is an important and powerful method in operator theory and has found deep and important applications in the theory of function spaces, partial differential equations, the theory of Fourier series, approximation theory, and computational mathematics. This method is based on Marcinkiewicz's interpolation theorem.

The report addresses the issue of boundedness for an important class of linear operators – integral operators.

We study the question of necessary and sufficient conditions for the boundedness of an integral operator in Lebesgue spaces Lp for all .

In terms of the kernel of the integral operator, necessary and sufficient conditions are obtained for the operator to be bounded in the Lebesgue space Lp for all . Using interpolation methods, new inequalities of Hardy-Littlewood type are obtained for two-dimensional generalized Lorentz spaces. New inequalities of the Nursultanov type are obtained for two-dimensional generalized Lorentz spaces.

Let  be a measurable space. Let the measure  have its continuity i.e. for any measurable set  and  exists  that .

The space  is the collection of all those measurable functions  satisfying



The distribution of a measurable function  on  is defined by



Then  is the decreasing rearrangement of .

Let  and  The Lorentz space  (see. [1]) is defined by those measurable functions  such that

when  and

 (1)

when  and



The Marcinkiewicz-Calderon interpolation theorem [1, 2] is well known.

Theorem. Let    If  is a quasi-linear operator and the constants ,  are such that

**** (2)

**** (3)

Then true

**** (4)

where  

For integral operators of the form

 (5)

From the papers [3]–[5] it follows that for 



Thus, for the integral operators, the conditions (2) and (3) in the interpolation theorem can be replaced by

 (6)

 (7)

Note that, the conditions (2), (3) and, accordingly (6), (7) are not necessary for the inequalities (4) to hold all 

In this report, we consider the problem of obtaining the necessary conditions in terms of the kernel of the operator (5) to satisfy (4) for all  The operators we study play a very important role in harmonic analysis.

Unlike the classical extrapolation problem ([3], [4], [5], [6], [7]), here we consider a narrower class of operators – integral operators and conditions are sought in terms of the kernal of the operator, and not in terms of the operators. I.e. this is the inverse problem to the Marcinkiewicz-Calderon interpolation theorem for integral operators with the conditions (6), (7).

For 2020, an interim report on research work “Interpolation theorems of the Marcinkevich-Calderon type and their applications” was prepared on the topic, inventory number of the report for 2020: 0220RK01665.

**MAIN PART OF THE REPORT ON SRW**

**1  Necessary conditions for the boundedness of the integral operator in Lp**

* 1. **Basic lemmas**

In this subsection, we describe the Lebesgue and Lorentz spaces, and also provide auxiliary lemmas to prove the main assertions.

Let  be a measurable space. Let the measure  have its continuity i.e. for any measurable set  and  exists  that .

The space  is the collection of all those measurable functions  satisfying



The distribution of a measurable function  on  is defined by



Then  is the decreasing rearrangement of .

Let  and  The Lorentz space  (see. [1]) is defined by those measurable functions  such that

when  and

 (1)

when  and



We define



We will use for  representation



Lemma 1.1.1. Let  be a locally integrable function. Then for any set of positive measure , there exists a set  such that 



Proof of lemma 1.1.1. For an arbitrary  such that , we define the following sets



Then

.

For definiteness, let

.

There are two cases:  and . In the first case, by the measure continuty property, there exists  such that  and

.

In the second case,  there are  such that  . The sign  is constant on , and we have



.

Let  be a realizing a minimum. Since , taking into account the continuity of the measure, there is a measurable  such that . Let , then





.

Thus we have

.

Lemma 1.1.2.Let  and the function  such that



there is a measurable locally integrable function. Then there is an estimate



where



Proof of lemma 1.1.2. Lemma 1.1.1 implies



,

where  is the non-increasing permutation of the function . We use lemma 1.1.1 again

.

Thus, we obtain

.

Lemma 1.1.3. ([5])Let ** and  Then we have



**1.2 Necessary conditions for the boundedness of the integral operator in Lp**

In this subsection we study the problem of obtaining the necessary conditions in terms of the kernel of the operator for the operator to be bounded in the Lebesgue space Lp for all .

For an integral operator of the form



From the papers [8]–[10] it follows that for 



The following theorem is true.

Theorem 1.2.1.Let  If for the integral operator



and for any  we have the relation



then





The proof is based on the lemmas given in the first section.

Proof of the Theorem 1.2.1. From the embeddings  and lemma 1.1.3 for any  we have

.

From the conditions of the theorem we obtain



for any .

Let  be arbitrary measurable sets of positive measure

. (8)

Put  choose  as follows. If



then select the parameter :



if



then



Thus we have 

Therefore, in the first case, from (8) we have



or





In the second case





Let now  then taking , we get





The second relation is proved similarly.

1. **Sufficient conditions for the boundedness of the integral operator in Lp**

In this subsection, we study the problem of obtaining sufficient conditions in terms of the kernel of the operator for the operator to be bounded in the Lebesgue space Lp for all .

Theorem 2.1.1. Let



If



then for arbitrary 



We say that a measure  is continuous if, for any - measurable set , there is a measurable subset  such that .

In what follows, we will assume that the measures  and  are continuous.

Theorem 2.1.2. Let   , 

, , .

If for the operator kernel 



takes place





Then the operator  is bounded from  to  and true the following inequlity

, ,

where 

Proof of the theorem 2.1.1. Let the conditions of the theorem be satisfied and let . Then we have the estimate







Applying Lemma 1.1.2, we obtain the following estimate





where

.

We estimate the function  as follows for 





and when 



Therefore, we have





We estimate each functional  First, note that

.

Hence,



.

Here we will assume that . By applying Hardy's inequality, in addition, in the second integral also applying Hardy's inequality twice, we obtain

.

Also, by applying Hardy's inequality twice, we obtain

.

Thus, we have

.

Remark 2.1.1 In [11], [12], [13], anisotropic spaces are considered and their interpolation properties are investigated by the interpolation method of Fernandez [14], [15], [11], [16]. The Marcinkiewicz-Calderon type theorems are obtained. These methods in the mentioned papers can be applied to obtain similar results in anisotropic spaces.

1. **Hardy-Littlewood type inequalities for two-dimensional generalized**

**Lorentz space**

This section deals with two-dimensional generalized Lorentz spaces. New inequalities of Hardy-Littlewood type are obtained for two-dimensional generalized Lorentz spaces.

Let  Two-dimensional generalized Lorentz spaces are defined as follows:



where  is the non-increasing permutation of the function .

If , then



If , then

.

Let .

.

If , then

.

If , then

.

Let  and  is the nonnegative function on . Define the classes of the function ,  and  as follows:

 is an increascing function,

and  is a decreasing function}.

 is an increascing function,

and  is a decreasing function }.

 is an increascing function,

and  is a decreasing function }.

The classes , B and C are defined as follows:

,  and .

Lemma 3.1.1. Let  and . Then the following inequality true

. (9)

Proof of Lemma 3.1.1 of inequality (9). Consider





By applying Hölder's inequalities and taking into account that  we get











Taking into account that  is an increasing function, we obtain the following estimate



Taking into account that  we obtain the following estimat





Lemma 3.1.2. Let  and . Then the inequalities are true

. (10)

Proof of Lemma 3.1.2 of inequality (10). Consider





By applying Hölder's inequalities, we obtain











Taking into account that  is a decreasing function, we obtain the following estimate







Let  and  Consider the Hardy and Bellman transforms

,



Lemma 3.1.3. Let 



If , then for any  there is such a representation of the function



such that



where  is the composition of operations:

for is the Hardy type transformation;

for  is the Bellman type transformation.

Proof of Lemma 3.1.2. Let be  Consider



Let  is the characteristic function of the set



where  is the measurable subset 



such a set can always be selected, since for a fixed 



Let  and, denote the functions

,

.

The each function  and  represent in the form , .

Let

,

where

.

,

where



Then





Thus, the representation is constructed



such that



Let

 is the Fourier transform of a function .

Theorem 3.1.1. Let  and   Then





Proof of Theorem 3.1.1. Consider







By applying Lemma 3.1.1, we obtain





By applying Lemma 3.1.3, we obtain

****

where









By using Lemma 3.1.2, we obtain the desired estimate.

**4  Nursultanov-type inequalities for two-dimensional generalized Lorentz spaces**

In this section, new inequalities of the Nursultanov type are obtained for two-dimensional generalized Lorentz spaces.

Let



Fourier transform of a function 

The inequalities connecting the integral properties of a function and its Fourier transform are well known.

Let   and  then the inequalities hold





where  is the classical Lorentz space. These inequalities are called the Hausdorff-Young and Hardy-Littlewood-Stein inequalities, respectively [17]. There are similar inequalities for Fourier transform on the interval  i.е. for , where are the Fourier transform with respect some orthonormal system.

The inequalities of the Hausdorff-Young (Hardy-Littlewood)-Stein type for the Fourier transform with respect to the regular system were obtained by E.D. Nursultanov in papers [10], [18]:



These statements are more general than the statements considered above, since the regular system is more general than all trigonometric systems, the Walsh system and the Price system with a bounded generator.

In [19] upper and lower bounds for the norm of the Fourier transform in generalized Lorentz spaces were obtained 

Theorem [19]. Let  and  belongs to the class  Then



where and  

The inequalities for the Fourier transform of functions from the generalized Lorentz space  The theorem is true giving upper and lower bounds for the norm of the Fourier transform of a function from to generalized Lorentz spaces 

If  then



Note that this inequality for the Fourier transform also follows from the results of [10], [18], where methods of network spaces are used.

The purpose of this subsection is to obtain an analogue of the Nursultanov inequality for the two-dimensional generalized Lorentz space.

The following theorem is true.

Theorem 4.1.1. Let  and   Then





where .

**CONCLUSION**

Necessary and sufficient conditions for the boundedness of the integral operator in the Lebesgue spaces Lp for all , new inequalities of Hardy-Littlewood type for two-dimensional generalized Lorentz spaces, new inequalities of Nursultanov type for two-dimensional generalized Lorentz spaces.

During the implementation of the project, the following results were obtained:

1. In terms of the kernel of the integral operator, necessary conditions are obtained for the operator to be bounded in the Lebesgue space Lp for all .

2. In terms of the kernel of the integral operator, sufficient conditions are obtained for the operator to be bounded in the Lebesgue space Lp for all ;

3. New inequalities of Hardy-Littlewood type are obtained for two-dimensional generalized Lorentz spaces.

4. New inequalities of the Nursultanov type are obtained for two-dimensional generalized Lorentz spaces.

The results of the work are of a theoretical nature and can be applied in harmonic analysis, the theory of functional spaces and equations of mathematical physics, and can also be used in scientific centers: KazNU named after Al-Farabi, Institute of Mathematics and Mathematical Modeling, Moscow State University named after M.V. Lomonosov, L.N. Gumilyov, E.A. Buketov, Steklov Mathematical Institute of the Russian Academy of Sciences and others.

3 papers have been accepted for publication, including 2 papers in the Eurasian Mathematical Journal and 1 paper in Mathematical Notes.

The results were tested at scientific seminars of the Kazakhstan branch of Moscow State University and the Institute of Mathematics and Mathematical Modeling.

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**APPENDIX А**

**List of publications**

1. Nursultanov E.D., Tleukhanova N.T. Interpolation and exstrapolation properties of integral operators // submitted to the editorial office in Eurasian Mathematical Journal. (Web of Science; Scopus SJR=0.277, Percentile 25 in category General Mathematics).
2. Bashirova A.N., Nursultanov E.D. On the inequality of different metrics for multiple Fourier-Haar series // accepted for publication in Eurasian Mathematical Journal. – 2021. – Vol. 12, № 3. – P. 90-93. (Web of Science; Scopus SJR=0.277, Percentile 25 in category General Mathematics).
3. Nursultanov E.D., Tleukhanova N.T., Mukeyeva Zh.M. On the Marcinkiewicz-Calderon interpolation theorem for integral operators // accepted for publication in Mathematical Notes. (Web of Science IF=0.673, Quratil Q4 in category Mathematics; Scopus SJR=0.723, Percentile 49 in category General Mathematics).

**APPENDIX B**

**Calendar work plan**

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Appendix 1.5

to this agreement

№\_\_ date «\_\_\_»\_\_\_\_\_\_\_2020 year

**CALENDAR WORK PLAN**

**1. Republican state enterprise on the right оf economic management**

**«Institute of mathematics and mathematical modeling»**

**Committee of Science of Ministry of Education**

**and Science of the Republic of Kazakhstan**

1.1 By priority: 8. Scientific research in the field of natural sciences.

1.2 By sub-priority: 8.1 Fundamental and applied research in the field of mathematics and mechanics.

1.3 On the topic of the project: **№ AP08956157** «Interpolation theorems of the Marcinkiewicz-Calderon type and their applications».

1.4 The total amount of the project amount 4 996 406 *(four million nine hundred ninety six thousand four hundred six*)tenge, including with a breakdown by years, for the performance of work in accordance with clause 3:

- for 2020 - in the amount of 2 999 126 (*two million nine hundred ninety nine thousand one hundred twenty six*) tenge;

- for 2021 - in the amount of 1 997 280 (one million nine hundred ninety seven thousand two hundred eighty) tenge.

**2. Characteristics of scientific and technical products by qualification**

**characteristics and economic indicators**

**2.1** Direction of work: harmonic analysis, theory of functional spaces.

**2.2** Applications: operator theory, approximation theory, theory of differential equations.

**2.3** Final result:

- for 2020 year: In terms of the kernel of the integral operator, to obtain the necessary conditions for the operator to be bounded in the Lp Lebesgue space for all ;

- for 2021 year: In terms of the kernel of the integral operator, to obtain sufficient conditions for the operator to be bounded in the Lp Lebesgue space for all . New inequalities of Hardy-Littlewood type for two-dimensional generalized Lorentz spaces will be obtained. New Nursultanov inequalities in two-dimensional generalized Lorentz spaces will be obtained.

As a result of the implementation of this scientific project for the entire period, the following will be published:

* at least 1 (one) paper or review published in a peer-reviewed scientific publication on the scientific direction of the project, included in 1 (first), 2 (second) or 3 (third) quartiles in the Web of Science database and (or) having a percentile according to Cite Score in the Scopus database, at least 50 (fifty), or in print in these publications;
* as well as at least 1 (one) paper in a peer-reviewed foreign and (or) domestic publication with a non-zero impact factor (recommended by CCSES MES RK).

**2.4** Patentability: not patentable.

**2.5** Scientific and technical level (novelty): high.

**2.6** The use of scientific and technical products is carried out: By the performer.

**2.7** Type of use of the result of scientific and (or) scientific and technical activities: theoretical.

**3. Name of work, terms of their implementation and results**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Task code, stage | | Name of work under the Agreement and the main stages of its implementation | Deadline for completion | | | Expected Result |
| start | | the ending |
| **2020 year** | | | | | | |
| 1 | | Necessary conditions for the operator to be bounded in Lebesgue space Lp for all  . | October 2020 | | December  2020 | In terms of the kernel of the integral operator, to obtain the necessary conditions for the operator to be bounded in the Lebesgue space Lp for all . |
| **2021 year** | | | | | | |
| 2 | | Sufficient conditions for the integral operator to be bounded in the Lebesgue space Lp for all . | January 2021 | | until September 30, 2021 | In terms of the kernel of the integral operator, sufficient conditions will be obtained for the operator to be bounded in the Lebesgue space Lp for all . |
| 3 | | Hardy-Littlewood type inequalities in weighted spaces | January 2021 | | until June 30, 2021 | New Hardy-Littlewood-type inequalities for two-dimensional generalized Lorentz spaces will be obtained. |
| 4 | | Nursultanov type inequalities in weighted spaces | July 2021 | | September2021 | New Nursultanov type inequalities will be obtained for two-dimensional generalized Lorentz spaces.  Based on the results of the implementation of this scientific project for the entire period, the following will be published: |
|  | |  |  | |  | * at least 1 (one) paper or review published in a peer-reviewed scientific publication on the scientific direction of the project, included in 1 (first), 2 (second) or 3 (third) quartiles in the Web of Science database and (or) having a percentile according to Cite Score in the Scopus database, at least 50 (fifty), or in print in these publications;   as well as at least 1 (one) paper in a peer-reviewed foreign and (or) domestic publication with a non-zero impact factor (recommended by CCSES MES RK). |
|  | | | | | | |
| By customer:  Chairman  GA « Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan»  \_\_\_\_\_\_\_\_\_\_\_\_\_\_ Kurmangaliyeva Zh.D.  l.s. | | | From the Performer:  General director of RSE on the REM «IMMM»  «Institute of mathematics and mathematical modeling» Committee of Science of Ministry of Education and Science of the Republic of Kazakhstan  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ M.A. Sadybekov  l.s.  Familiarized:  Нead of research work  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Kopezhanova A.N. | | | |