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**РЕФЕРАТ**

Есеп беру жұмысы34 б., 34 әдебиет көздері, 2 қосымшалар.

ГРИН ФУНКЦИЯСЫ, РИМАН-ГРИН ФУНКЦИЯСЫ, СИПАТТАМАЛЫҚ ҮШБҰРЫШ, ЛОКАЛЬДЫ ЕМЕС ШЕКАРАЛЫҚ ЕСЕП, ТОЛҚЫН ТЕҢДЕУІ, ЕКІ ӨЛШЕМДІ ГИПЕРБОЛАЛЫҚ ТЕҢДЕУ

Жобаның мақсаты сипаттамалық үшбұрыштағы гиперболалық теңдеу үшін асиметриялы сипаттамалық бастапқы шекаралық есептерінің Грин функциясын құру әдісін негіздеу.

Есептің негізгі ғылыми нәтижелері күнтізбелік кестеге сәйкес алынған:

* ширек жазықтықтағы жалпы түрдегі екінші ретті екі өлшемді ги-перболалық теңдеу үшін бірінші бастапқы шекаралық есептің Грин функциясы құрылды;
* ширек жазықтықтағы жалпы түрдегі екінші ретті екі өлшемді ги-перболалық теңдеу үшін екінші бастапқы шекаралық есептің Грин функциясы құрылды;
* сипаттамалық емес шекарада бірінші текті шекаралы шартты сипаттамалық үшбұрышта қарастырылатын жалпы түрдегі гиперболалық теңдеу үшін асиметриялы сипаттамалық шекаралық есептердің Грин функциясы құрылды;
* сипаттамалық емес шекарада екінші текті шекаралық шартты сипаттамалық үшбұрышта қарастырылатын жалпы түрдегі гиперболалық теңдеу үшін асимметриялы сипаттамалық шекаралық есептердің Грин функциясы құрылды;
* грин функциясының «классикалық емес» түріндегі дұрыс сипаттамалық шекаралы есеп мысалы құрастырылды.

Жобада қойылған мәселелердің орындалу деңгейі. Жобаның күнтізбелік жоспарында қарастырылған мәселелердің барлығы орындалды, алға қойылған мақсаттарға қол жеткізілді. Жобаның негізгі нәтижелерін алу барысында туындаған қосымша ғылыми нәтижелер алынған және жарияланған (Web of Science және / немесе Scopus деректер базасына енген халықаралық ғылыми журналдарда) жобаның нәтижелерінің жоспарланған кестесіне қосымша.

Ғылыми жариялымдар. Зерттеу нәтижелері бойынша 2021 жылдың мамыр айынан бастап жоба қызметкерлері 2 ғылыми мақалаларды, соның ішінде:

* импакт-факторы бар Web of Science Q1, Scopus процентиль 83 мәліметтер базасына енгізілген халықаралық рейтингтік журналда 1 мақала (Article);
* ҚР БҒМ ҚОКСОН ұсынған қазақстандық журналда 1 мақала.

**ABSTRACT**

Report 34 p., 31 sources, 2 appendices.

GREEN’S FUNCTION, RIEMANN-GREEN FUNCTION, CHARACTERISTIC TRIANGLE, NONLOCAL BOUNDARY VALUE PROBLEM, WAVE EQUATION, TWO-DIMENSIONAL HYPERBOLIC EQUATION

Project goal is to justify the method of the Green’s function for asymmetric characteristic initial boundary value problems for a hyperbolic equation in a characteristic triangle.

The main scientific results of the report obtained according to the calendar plan:

* the Green’s function for a first initial boundary value problem in a quarter of the space for a general two-dimensional hyperbolic equation of the second order is constructed;
* the Green’s function for a second initial boundary value problem in a quarter of the space for a general two-dimensional hyperbolic equation of the second order is constructed;
* the Green’s function for asymmetric characteristic boundary value problems for a hyperbolic equation of the general form considered in a characteristic triangle with a boundary condition of the first kind on a non-characteristic boundary is constructed;
* the Green’s function for asymmetric characteristic boundary value problems for a hyperbolic equation of the general form considered in a characteristic triangle with a boundary condition of the second kind on a non-characteristic boundary is constructed;
* an example of a well-posed characteristic boundary value problem having a "non-classical" form of the Green’s function is constructed.

The degree of fulfillment of the tasks set in the project. All the tasks set in the project calendar plan have been completed, all the planned goals have been achieved.

Scientific publications. According to the research results, since May 2021, the project staff has published 2 scientific articles, including:

- 1 article in an international rating journal included in the Web of Science Q1 database, Scopus percentile 83;

- 1 article in journal recommended by CQAEA of MES of RK.

**CONTENT**

[INTRODUCTION 6](#_Toc54887669)

[MAIN PART OF THE RESEARCH WORK REPORT 7](#_Toc54887670)

[1 Construction of the Green’s function for a first initial boundary value problem in a quarter of the space for a general two-dimensional hyperbolic equation of the second order 7](#_Toc54887671)

[2 Construction of the Green’s function for a second initial boundary value problem in a quarter of the space for a general two-dimensional hyperbolic equation of the second order 11](#_Toc54887674)

[3 Construction of the Green’s function for asymmetric characteristic boundary value problems for a hyperbolic equation of the general form considered in a characteristic triangle with a boundary condition of the first kind on a non-characteristic boundary 14](#_Toc54887677)

[4 Construction of the Green’s function for asymmetric characteristic boundary value problems for a hyperbolic equation of the general form considered in a characteristic triangle with a boundary condition of the second kind on a non-characteristic boundary 17](#_Toc54887678)

[5 Construction of an example of a well-posed characteristic boundary value problem having “non-classical” form of the Green’s function 2](#_Toc54887679)1

[CONCLUSION 24](#_Toc54887680)

[LIST OF USED SOURCES 25](#_Toc54887681)

[APPENDIX А - List of published works 28](#_Toc54887682)

[APPENDIX B - Calendar plan of works](#_Toc54887683) 29

**INTRODUCTION**

Content of the report.This final report (for twelve months of the project) presents the results of research on the definition and construction of Green's functions of boundary value problems for a general second-order hyperbolic equation.

The significance of the project lies in the fact that the objects under study - Green's functions for boundary value problems for linear differential operators with local and nonlocal conditions, on one hand, are of great importance in mathematical science itself, from a purely mathematical point of view. On the other hand, there is significant interest in such problems in mechanics, physics, biology and other natural science disciplines. Therefore, the results obtained are relevant and will be understandable to scientists all over the world. And they can be used by them for further research.

The principal difference between the ideas of the Project is that it is devoted to the study of Green's functions for hyperbolic problems. There has been very little research in this direction and for a long time there have been no new advances. The completed project differs from the works of other authors of past years in that the investigated problems are not symmetric. The proposed project differs from the project manager's own works in that earlier, for similar tasks, he considered the issues of correctness and spectral properties, and the issues of constructing Green's functions were not considered.

During the implementation of the project, various mathematical sources have been used, both monographs and journal articles [1-34]. The sources used are listed in the section at the end of the report.

Appendix A lists the published works of the authors of the report, and Appendix B contains the calendar plan for 2021.

**MAIN PART OF THE RESEARCH WORK REPORT**

1 Construction of the Green's function for the first initial-boundary value problem in a quarter-plane for a general two-dimensional second-order hyperbolic equation

In this section, according to the expected result of the contract schedule, a definition is given and a method for constructing the Green's function for the first initial-boundary value problem in a quarter-plane for a general two-dimensional hyperbolic equation of the second order is substantiated.

In the domain , the linear hyperbolic equation

(1.1)

is considered. For the general second-order hyperbolic equation (1.1), we consider the first initial boundary value problem

(1.2)

(1.3)

where

.

Despite the seeming simplicity of the domain, it has turned out that it is necessary to carry out detailed research and give precise definitions.

Theorem 1.1. Let

. Then problem (1.1)-(1.3) has a unique regular solution.

Thus, first of all, we have proved the correctness of the considered problem (1.1)-(1.3) in the classical sense.

It is well known that the Riemann-Green function is not defined in the entire domain , and only for those points that satisfy the following conditions: . And for the rest of the points of the domain, the Riemann-Green function is not uniquely determined. For further constructions, it is important for us to use the Riemann-Green function defined at all points of the domain , including at .

Also, for further reasoning, we need to introduce some relations between the coefficients and on the border .

To introduce the Riemann-Green function at all points of the domain , we continue the coefficients of equation (1.1) into the domain as follows:

(1.4)

(1.5)

(1.6)

If the coefficients , then from (1.4)-(1.6) the coefficients in the domain have the smoothness and satisfy the following symmetry conditions:

(1.7)

Lemma 1.1.If the conditions (1.7) hold, then the Riemann-Green function of equation (1.1) has the following symmetry:

(1.8)

In this project, for the first time, we have given the definition of the Green's function of this problem.

Definition 1.1. The Green's function of problem (1.1)-(1.3) is the function , which for every fixed , satisfies the homogeneous equation

(1.9)

and the following boundary conditions:

(1.10)

(1.11)

(1.12)

and on the above characteristic lines, the following conditions must hold: the values of the derivatives of the Green's function in directions parallel to these characteristics must coincide in adjacent domains; i.e.,

(1.13)

(1.14)

(1.15)

and the "corner condition":

(1.16)

must be satisfied as the regions meet at .

Theorem 1.2. Function that satisfies the conditions (1.9)-(1.16) exists and is unique.

To show that such a function that satisfies the conditions (1.9)-(1.16), exists and is unique, we divide the domain into eight subdomains . From the course of the proof of this theorem, we obtain the following result:

Corollary 1.1. The Green's function of problem (1.1)-(1.3) in subdomains has the form:

In order to construct the Green's function in the subdomain , we assume that (1.7) hold and continue the coefficients to . If (1.7) hold, then we have the symmetry of the Riemann-Green function (1.8), and using this function we can write the explicit form of the Green's function in the following form:

Further, using the constructed Green's function, an integral representation of the solution to problem (1.1)-(1.3) is obtained.

Work on this item of the schedule has been fully completed. The main result was published in [33] in the journal “Boundary Value Problems" (Web of Science Q1, Scopus percentile 83).

2 Construction of the Green's function for the second initial-boundary value problem in a quarter-plane for a general two-dimensional second-order hyperbolic equation

In this section, according to the expected result of the schedule of the contract, we give the definition and justification of the method for constructing the Green's function for the second initial-boundary value problem in a quarter of the plane for a general two-dimensional hyperbolic equation of the second order.

In the domain , the linear hyperbolic equation

(2.1)

is considered. For the general second-order hyperbolic equation (2.1), we consider a second initial-boundary value problem

(2.2)

(2.3)

where .

Theorem 2.1. Let

. Then problem (2.1)-(2.3) has a unique regular solution.

Thus, as well as for the first initial-boundary value problem, first of all, the correctness of the considered problem (2.1)-(2.3) is proved in the classical sense.

As in the first problem, in order to determine the Riemann-Green function defined at all points of the domain , at , we use the conditions (1.7).

Definition 2.1. The Green's function of problem (2.1)-(2.3) is the function , which for every fixed , satisfies the homogeneous equation

(2.4)

and the following boundary conditions:

(2.5)

(2.6)

(2.7)

and on the above characteristic lines, the following conditions must hold: the values of the derivatives of the Green's function in directions parallel to these characteristics must coincide in adjacent domains; i.e.,

(2.8)

(2.9)

(2.10)

and the "corner condition":

(2.11)

must be satisfied as the regions meet at .

Theorem 2.2. Function which satisfies the conditions (2.4)-(2.11), exists and is unique.

To show that such a function that satisfies the conditions (2.4)-(2.11), exists and is unique, we divide the domain into eight subdomains . From the course of the proof of this theorem, we obtain the following result:

Corollary 2.1. The Green's function of problem (2.1)-(2.3) in subdomains has the form:

As in the first initial-boundary value problem, in order to construct the Green's function in the domain , we assume that (1.7) hold and we continue the coefficients to . If (1.7) hold, then we have the symmetry of the Riemann-Green function (1.8), and using this function we can write the explicit form of the Green's function in the following form:

Further, using the constructed Green's function, an integral representation of the solution to the problem (2.1)-(2.3) is obtained.

Work on this item of the schedule has been fully completed.

3 Construction of the Green's function for asymmetric characteristic boundary value problems for a general hyperbolic equation considered in a characteristic triangle with a boundary condition of the first kind on a non-characteristic boundary

In this section, according to the expected result of the schedule of the contract, the definition of the Green's function for asymmetric characteristic boundary value problems for a general hyperbolic equation in a characteristic triangle with a boundary condition of the first kind on a non-characteristic boundary is given and the substantiation of the method of its construction is given.

In the characteristic triangle, a general second-order hyperbolic equation

(3.1)

is considered. For equation (3.1), we consider a non-local boundary value problem with displacement

(3.2)

where is a given constant. On the non-characteristic line , a boundary condition of the first kind is set

(3.3)

We will assume that

Using the representation of the solution to the Cauchy problem (3.1), (3.3) by the Riemann method, we seek the solution to problem (3.1)-(3.3) in the following form:

where is the Riemann-Green function of equation (3.1). Then substituting (3.4) into (3.2), we have:

where

Then equation (3.5) will be a Fredholm equation of the second kind. Therefore, if we assume that the solution to problem (3.5) is unique, then it exists.

This proves that the considered problem (3.1)-(3.3) is Fredholm.

Definition 3.1. The Green's function of problem (3.1)-(3.3) is the function , which for every fixed , satisfies the homogeneous equation

(3.6)

and the following boundary conditions:

(3.7)

(3.8)

and on the above characteristic lines, the following conditions must hold: the values of the derivatives of the Green's function in directions parallel to these characteristics must coincide in adjacent domains; i.e.,

(3.9)

(3.10)

(3.11)

(3.12)

and the "corner condition":

(3.13)

must be satisfied as the regions meet at .

The definition of the Green's function of problem (3.1)-(3.3) differs significantly from the definition of the Green's functions of the previous two problems in that here the Green's function has jumps on two more additional characteristics.

In order to construct the function that satisfies conditions (3.6)-(3.13), we have selected six subdomains of the domain . In each of these subdomains, the Green's function is continuous, but when passing from one domain to another, the continuity of the Green's function may be broken.

Theorem 3.1. If the solution to problem (3.1)-(3.3) is unique, then the function that satisfies the conditions (3.6)-(3.13), exists and is unique.

Initially, we have constructed the Green's function for a differential expression of the form

In what follows, this technique is extended to the Green's function for the general equation (3.1).

The work on this section of the schedule has been fully completed. The main result when was published in our work [34] in the journal “*KazNU Bulletin. Series mathematics, mechanics, computer science*" (recommended by CQAEA of MES of RK).

4 Construction of the Green's function for asymmetric characteristic boundary value problems for a general hyperbolic equation considered in a characteristic triangle with a boundary condition of the second kind on a non-characteristic boundary

In this section, according to the expected result of the schedule of the contract, a definition of the Green's function for asymmetric characteristic boundary value problems for a general hyperbolic equation in a characteristic triangle with a boundary condition of the second kind on a non-characteristic boundary is given and a substantiation of the method of its construction is given.

In the characteristic triangle , a general second-order hyperbolic equation

(4.1)

is considered. For equation (4.1), we consider a non-local boundary value problem with displacement

(4.2)

where is a given constant. On the non-characteristic line , a boundary condition of the second kind is set

(4.3)

We will assume that

As in Section 3, using the representation of the solution to the Cauchy problem (4.1), (4.3) by the Riemann method, we seek the solution of problem (4.1)-(4.3) in the following form:

where is the Riemann-Green function of equation (4.1). Then, substituting (4.4) into (4.2), we obtain:

(4.5)

where

Then equation (4.5), as in the previous section, will be the Fredholm equation of the second kind. If we assume that the solution to problem (4.5) is unique, then it exists.

This proves that the considered problem (4.1)-(4.3) is Fredholm.

Definition 4.1. The Green's function of problem (4.1)-(4.3) is the function , which for every fixed , satisfies the homogeneous equation

(4.6)

and the following boundary conditions:

(4.7)

(4.8)

and on the above characteristic lines, the following conditions must hold: the values of the derivatives of the Green's function in directions parallel to these characteristics must coincide in adjacent domains; i.e.,

(4.9)

(4.10)

(4.11)

(4.12)

and the "corner condition":

(4.13)

must be satisfied as the regions meet at .

As in the previous problem, in order to construct a function that satisfies conditions (4.6)-(4.13), we have selected six subdomains of the domain .

Theorem 4.1. If the solution to problem (4.1)-(4.3) is unique, then the function that satisfies the conditions(4.6)-(4.13), exists and is unique.

Initially, we have constructed the Green's function for a differential expression of the form

In what follows, this technique is extended to the Green's function for a general equation (4.1).

The work on this section of the schedule has been fully completed.

5 Construction of an example of a well-posed characteristic boundary value problem having a "nonclassical" form of the Green's function

In this section, according to the expected result of the contract schedule an example of a well-posed characteristic boundary value problem having a "nonclassical" form of the Green's function is constructed.

In the characteristic triangle , the wave equation

(5.1)

is considered. For equation (5.1), we consider boundary value problem

(5.2)

where is a given constant. On the non-characteristic line , a boundary condition of the first kind is set

(5.3)

It is easy to show that, when , problem (5.1)-(5.3) is well posed.

Such a problem for the case of a degenerate hyperbolic equation was first considered in the work of T. Sh. Kalmenov [32].

Further for the first time we give the definition of the Green's function of problem (5.1)-(5.3).

Definition 5.1. The Green's function of problem (5.1)-(5.3) is the function , which for every fixed , satisfies the homogeneous equation

(5.4)

and the following boundary conditions:

(5.5)

(5.6)

(5.7)

(5.8)

(5.9)

(5.10)

(5.11)

(5.12)

(5.13)

(5.14)

(5.15)

(5.16)

When or :

(5.17)

When or :

(5.18)

When or :

(5.19)

The definition of the Green's function of problem (5.1)-(5.3), as can be seen, differs significantly from the definition of the Green's functions of all previous problems in that the Green's function of this problem has jumps on eight characteristics.

Theorem 5.1. If , then the function that satisfies the conditions (5.4)-(5.19), exists and is unique.

In the proof of Theorem 5.1, the fifteen subdomains of the domain are singled out and the Green's function is sought in the following form:

As a result, we have obtained the Green's function of problem (5.1)-(5.3) explicitly and it has the following form:

It is easy to see from Definition 5.1 that the Green's function of problem (5.1)-(5.3) has "nonclassical" form, because it has discontinuities in eight characteristics.

The work on this section of the schedule has been fully completed.

**CONCLUSION**

Thus, in this final report for twelve months of the project, the results of investigations of the Green's function of boundary value problems for a general second-order hyperbolic equation are presented.

The central result of the report is the substantiation of the Green's function method for asymmetric characteristic initial-boundary value problems for a hyperbolic equation in a characteristic triangle. The main achievement of the report is the identification of a new effect: in contrast to Green's functions for elliptic and parabolic problems, if for problems for equations of elliptic and parabolic type, the Green's function can be represented as a sum of the "principal part with a singularity" and a "smooth summand" , then for hyperbolic boundary value tasks this is no longer the case. The function can have much more features and breaks than the "main part" . This fact significantly complicates the consideration and, therefore, a separate study is required for each separate case of boundary value problems.

The degree of novelty of the results obtained. All scientific results presented in the report are new.

The degree of fulfillment of the tasks set in the project. All tasks stipulated in the project schedule have been completed, all set goals have been achieved.

Completeness of results. The results stated and presented in the report have been fully proven. The completion of the project results is confirmed by the fact that the research results have been published in the foreign peer-reviewed scientific journal, indexed in the Web of Science Clarivate Analytics and Scopus.

Scientific publications. Based on the research results, since May 2021, the project staff published 2 scientific articles, including:

* 1 article (Article) in an international ranking journal included in the Web of Science Clarivate Analytics database and the Scopus database with an impact factor;
* 1 article in the domestic journal recommended by CQAEA of MES of RK.

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**APPENDIX A**

**List of published works**

Based on the research results, 2 scientific articles have been published since May 2021, including:

* *1 article (Article) in an international ranking journal included in the Web of Science Clarivate Analytics database and the Scopus database with an impact factor*;
* *1 article in domestic journals recommended by CQAEA of MES of RK.*

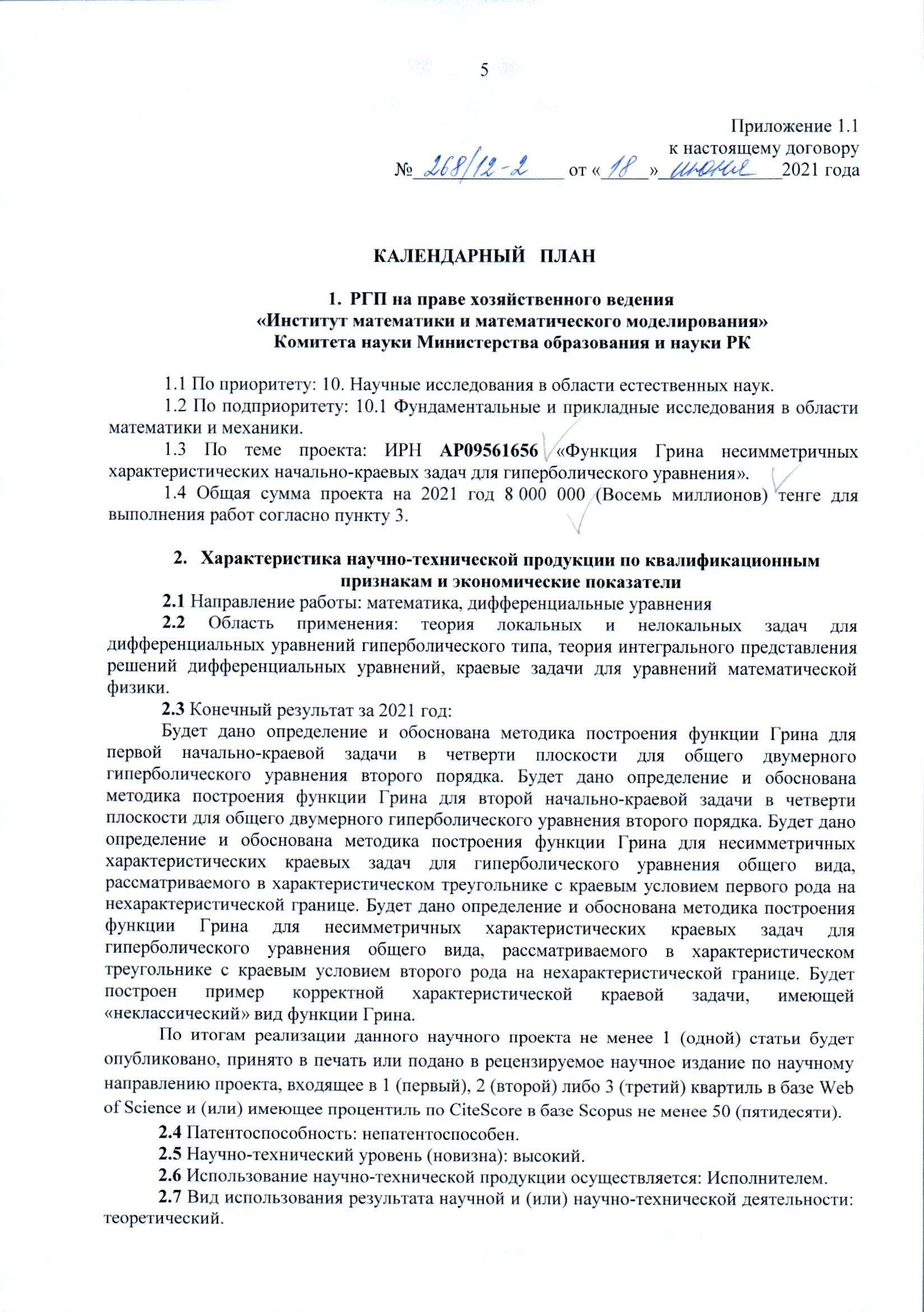
1 article in the international ranking journal (included in the Web of Science Clarivate Analytics database and Scopus database) with an impact factor:

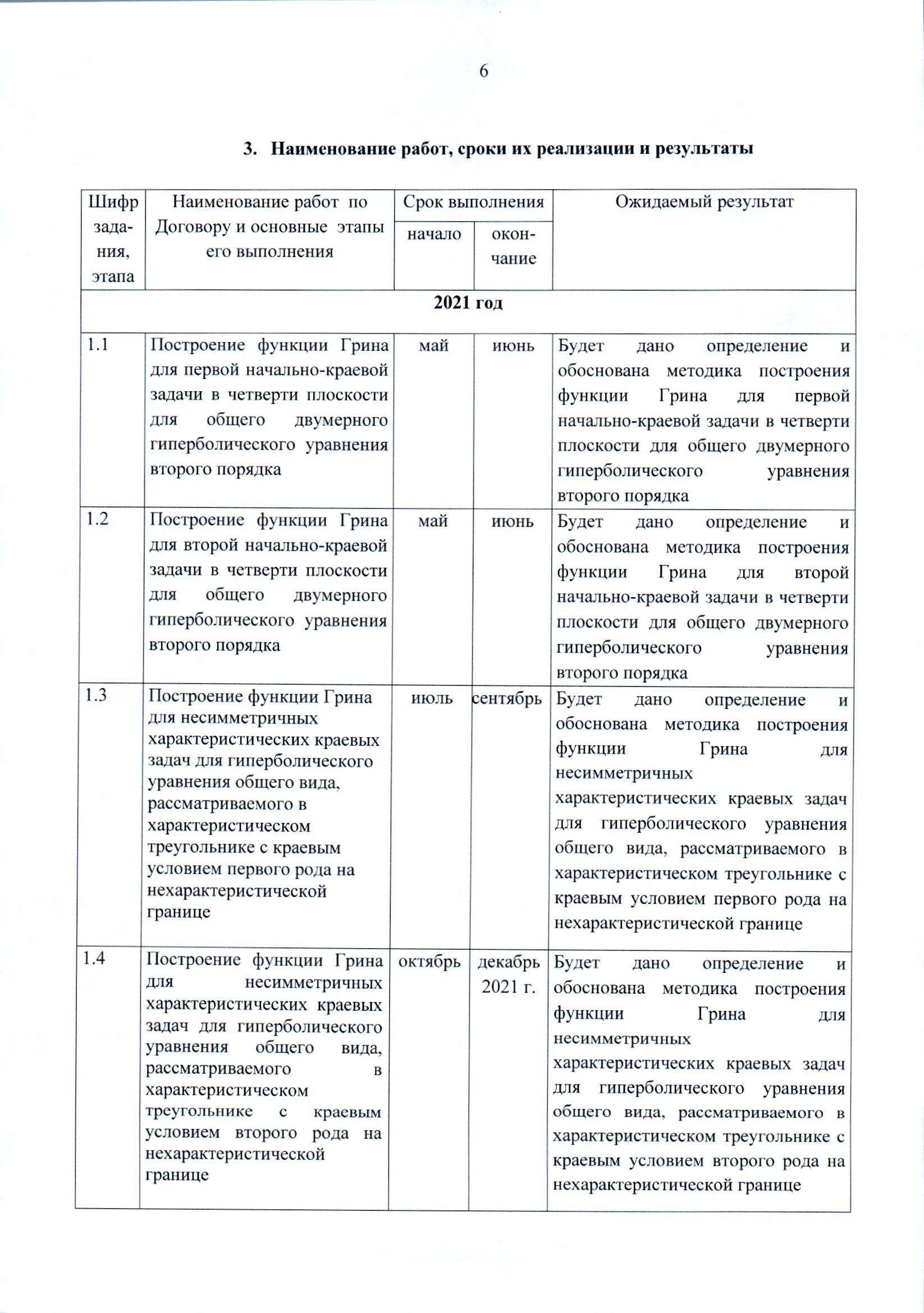
1. Sadybekov M. A., Derbissaly B. O. On Green's function of Cauchy – Dirichlet problem for hyperbolic equation in a quarter plane // Boundary Value Problems. - 2021. - V. 69, 23 pp., Web of Science Q1, Scopus percentile 83.

1 article in the domestic journal recommended by CQAEA of MES of RK.

2 Derbissaly B. O., Sadybekov M. A. On Green's function of Darboux problem for hyperbolic equation // Bulletin of KazNU. Series of mathematics, mechanics, computer science. - 2021. - V. 111, No. 3. - P. 79-94.

**APPENDIX B**

 **Calendar plan of works**





**Translation into English of the calendar plan of the grant project**

**" Green's function of asymmetric characteristic initial boundary value problems for a hyperbolic equation "**

Appendix 1.1

to Agreement № \_\_ from \_\_\_\_\_\_2021

for grant funding

**WORK SCHEDULE**

Under agreement № 268/12-2 from June 18, 2021

1. **RSE on the right of economic management “Institute of mathematics and mathematical modeling” of the Committee of science of the Ministry of education and science of the Republic of Kazakhstan**
   1. By priority: 10 Research in the field of natural sciences
   2. By sub-priority: 10.1 Fundamental and applied research in mathematics and mechanics
   3. On the theme of the project: **No. AP09561656** “Green's function of asymmetric characteristic initial boundary value problems for a hyperbolic equation”
   4. The total amount of the project for 2021 is 8 000 000 (eight million) tenge, for implementing the works according to item 3.
2. **Characteristics of scientific and technical products by qualification features and economic indicators**
   1. Direction of the work: mathematics, differential equation.
   2. Applications: theory of local and non-local problems for differential equations of hyperbolic type, theory of integral representation of solutions of differential equations, boundary value problems for equations of mathematical physics.
   3. Final result for 2021:

We will give a definition and justify a method for constructing the Green’s function for a first initial boundary value problem in a quarter of the space for a general two-dimensional hyperbolic equation of the second order. We will give a definition and justify a method for constructing the Green’s function for a second initial boundary value problem in a quarter of the space for a general two-dimensional hyperbolic equation of the second order. We will give a definition and justify a method for constructing the Green's function for asymmetric characteristic boundary value problems for a general hyperbolic equation in a characteristic triangle with a boundary condition of the first kind on a non-characteristic boundary. We will give a definition and justify a method for constructing the Green's function for asymmetric characteristic boundary value problems for a general hyperbolic equation in a characteristic triangle with a boundary condition of the second kind on a non-characteristic boundary. We will construct an example of a well-posed characteristic boundary value problem having a "non-classical" form of the Green's function

Based on the results of the implementation of this project, the following documents will be published:

- at least 1 (one) article or review published in a peer-reviewed scientific publication on the scientific direction of the project, included in 1 (first), 2 (second) or 3 (third) quartiles in the Web of Science database and (or) having a percentile according to CiteScore at least 50 (fifty) in the Scopus database, or printed in the indicated publications;

* 1. Patentability: not patentable.
  2. Scientific and technical level (novelty): high.
  3. The use of scientific and technical products is carried out: by the Executor.
  4. Type of use of the result of scientific and (or) scientific and technical activities: theoretical.

1. **Name of work, terms of their implementation and results**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Stage code | Name of works under the Contract and the main stages of its implementation | | | Deadlines | | | Expected results |
| Start | | Ending |
| **2021** | | | | | | | |
| 1.1 | Construction of the Green’s function for a first initial boundary value problem in a quarter of the space for a general two-dimensional hyperbolic equation of the second order | | | May | | June | We will give a definition and justify a method for constructing the Green’s function for a first initial boundary value problem in a quarter of the space for a general two-dimensional hyperbolic equation of the second order |
| 1.2 | | Construction of the Green’s function for a second initial boundary value problem in a quarter of the space for a general two-dimensional hyperbolic equation of the second order | May | | June | | We will give a definition and justify a method for constructing the Green’s function for a second initial boundary value problem in a quarter of the space for a general two-dimensional hyperbolic equation of the second order |
| 1.3 | | Construction of the Green’s function for asymmetric characteristic boundary value problems for a hyperbolic equation of the general form considered in a characteristic triangle with a boundary condition of the first kind on a non-characteristic boundary | July | | September | | We will give a definition and justify a method for constructing the Green’s function for asymmetric characteristic boundary value problems for a hyperbolic equation of the general form considered in a characteristic triangle with a boundary condition of the first kind on a non-characteristic boundary |
| 1.4 | | Construction of the Green’s function for asymmetric characteristic boundary value problems for a hyperbolic equation of the general form considered in a characteristic triangle with a boundary condition of the second kind on a non-characteristic boundary | October | | December | | We will give a definition and justify a method for constructing the Green’s function for asymmetric characteristic boundary value problems for a hyperbolic equation of the general form considered in a characteristic triangle with a boundary condition of the second kind on a non-characteristic boundary |
| 1.5 | | Construction of an example of a well-posed characteristic boundary value problem having “non-classical” form of the Green’s function | October | | December | | We will construct an example of a well-posed characteristic boundary value problem having “non-classical” form of the Green’s function.  Based on the results of the implementation of this scientific project,  at least 1 (one) article will be published, printed or submitted in a peer-reviewed scientific publication on the scientific direction of the project, included in 1 (first), 2 (second) or 3 (third) quartiles in the Web of Science database and (or) having a percentile according to CiteScore at least 50 (fifty) in the Scopus database. |

|  |  |
| --- | --- |
| From customer:  Chairman of the State Institution "Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan"  \_\_\_\_\_\_\_\_\_\_\_\_\_Kurmangalieva Zh.D.  l.s. | From the Contractor:  General Director of the RSE on the RK "Institute of Mathematics and Mathematical Modeling" of the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Sadybekov M.A.  l.s.  Familiarized with:  Scientific supervisor of the project  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Sadybekov M.A. |